Double negation, excluded middle and accessibility in dynamic semantics

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Puzzles of accessibility

First-generation dynamic semantic theories[1] were introduced in part to handle discourses such as (1), in which a pronoun is (apparently) outside the scope of its binder.

[1] E.g. file change semantics (FCS, Heim [1982, 1983]), discourse representation theory (DRT, Kamp [1981], Kamp and Reyle [1993], Kamp et al. [2011]), dynamic predicate logic (DPL, Groenendijk and Stokhof [1991]).

(1) John owns a car. It's parked in a weird place.

The standard move is to treat a sentence meaning as some kind of transition between variable assignments or related structures. As an example, Figure 1 gives the semantics of DPL, which has the same syntax as standard predicate logic (PL).

$$\begin{aligned}
[\![Pt_1 \ldots t_n]\!]^f_M &= \left\{g \mid f = g \;\&\; \left\langle [\![t_1]\!]^g_M, \ldots, [\![t_n]\!]^g_M \right\rangle \in \mathcal{I}(P)\right\} \\
[\![t_1 = t_2]\!]^f_M &= \left\{g \mid f = g \;\&\; [\![t_1]\!]^g_M = [\![t_2]\!]^g_M\right\} \\
[\![\neg\phi]\!]^f_M &= \left\{g \mid f = g \;\&\; [\![\phi]\!]^g_M = \varnothing\right\} \\
[\![\phi \wedge \psi]\!]^f_M &= \left\{h \mid \text{there's a } g : g \in [\![\phi]\!]^f_M \;\&\; h \in [\![\psi]\!]^g_M\right\} \\
[\![\phi \vee \psi]\!]^f_M &= \left\{g \mid f = g \;\&\; [\![\phi]\!]^g_M \cup [\![\psi]\!]^g_M \neq \varnothing\right\} \\
[\![\phi \rightarrow \psi]\!]^f_M &= \left\{g \mid f = g \;\&\; [\![\phi]\!]^g_M \subseteq \left\{h \mid [\![\psi]\!]^h_M \neq \varnothing\right\}\right\} \\
[\![\exists x\phi]\!]^f_M &= \left\{h \mid \text{there's a } g : f[x]g \;\&\; h \in [\![\phi]\!]^g_M\right\} \\
[\![\forall x\phi]\!]^f_M &= \left\{g \mid f = g \;\&\; \{h \mid g[x]h\} \subseteq \left\{h \mid [\![\phi]\!]^h_M \neq \varnothing\right\}\right\}
\end{aligned}$$

Figure 1: Semantics of DPL [Groenendijk and Stokhof, 1991, 54]

- Note that in Figure 1, in the clauses for predication, identity, negation, disjunction, implication and the universal quantifier the set of output assignments for input assignment f is either $\{f\}$ or $\varnothing$.

Formulae with this property are called **tests**.

- In DPL, (1) can be translated as shown in (2), in which the variable representing the pronoun really is outside the syntactic scope of the quantifier representing its binder. And yet, the bound effect is achieved, as shown.

(2) $\exists x(Cx \wedge Ojx) \wedge Px$

$$
\begin{aligned}
[\![(2)]\!]^f_M &= \left\{h \mid \text{ there's a } g : g \in [\![\exists x(Cx \land Ojx)]\!]^f_M \;\&\; h \in [\![Px]\!]^g_M\right\} \\
&= \left\{h \mid \text{ there's a } g,k : f[x]g \;\&\; k \in [\![Cx \land Ojx]\!]^g_M \;\&\; k = h \;\&\; h \in [\![Px]\!]^g_M\right\} \\
&= \{g \mid f[x]g \;\&\; g(x) \in \mathcal{I}(C) \;\&\; \langle \mathcal{I}(j), g(x)\rangle \in \mathcal{I}(O) \;\&\; g(x) \in \mathcal{I}(P)\}
\end{aligned}
$$

- In contrast, no binding is possible in the case of (3).

(3) John doesn't own a car. It's parked in a weird place.

(4) $\neg\exists x(Cx \land Ojx) \land Px$

$$
\begin{aligned}
[\![(4)]\!]^f_M &= \left\{h \mid \text{ there's a } g : g \in [\![\neg\exists x(Cx \land Ojx)]\!]^f_M \;\&\; h \in [\![Px]\!]^g_M\right\} \\
&= \left\{h \mid \text{ there's a } g : f = g \;\&\; [\![\exists x(Cx \land Ojx)]\!]^g_M = \emptyset \;\&\; g = h \;\&\; h \in [\![Px]\!]^g_M\right\} \\
&= \{g \mid f = g \;\&\; \{h \mid g[x]h \;\&\; h(x) \in \mathcal{I}(C) \;\&\; \langle \mathcal{I}(j), h(x)\rangle \in \mathcal{I}(O)\} = \emptyset \;\&\; g(x) \in \mathcal{I}(P)\}
\end{aligned}
$$

- This is a welcome result for (3).
- But since the closing off of anaphoric dependencies is tied to negation, it follows that $\neg\neg\phi$ is not generally equivalent to ϕ :

> [T]he law of double negation will not hold unconditionally. Consider a formula ϕ that is not a test. Negating ϕ results in the test $\neg\phi$, and a second negation, which gives $\neg\neg\phi$, does not reverse this effect [. . .] Hence, double negation is not in general eliminable.[2]

[2] [Groenendijk and Stokhof, 1991, 62]

This failure of double negation elimination is problematic,[3] given examples where it seems that we would like a doubly-negated existential statement to behave more like its un-negated counterpart than these theories predict.

[3] As has been noted e.g. by Groenendijk and Stokhof [1990, 1991], Kamp and Reyle [1993], Krahmer and Muskens [1995].

Double Negation

One class of examples concerns straightforward double negations, such as (5).

(5) It's not true that John doesn't own a car. It's (just) parked in a weird place.

(6) $\neg\neg\exists x(Cx \land Ojx) \land Px$

$$
\begin{aligned}
[\![(6)]\!]^f_M &= \left\{h \mid \text{ there's a } g : g \in [\![\neg\neg\exists x(Cx \land Ojx)]\!]^f_M \;\&\; h \in [\![Px]\!]^g_M\right\} \\
&= \{g \mid f = g \;\&\; \{h \mid g[x]h \;\&\; h(x) \in \mathcal{I}(C) \;\&\; \langle \mathcal{I}(j), h(x)\rangle \in \mathcal{I}(O)\} \neq \emptyset \;\&\; g(x) \in \mathcal{I}(P)\}
\end{aligned}
$$

Disjunction

Another class of examples concerns disjunctions like (7).

(7) Either John doesn't own a car, or it's parked in a weird place.

(8) $\neg\exists x(Cx \wedge Ojx) \vee Px$

$$[\![(8)]\!]^f_M = \Big\{g \mid f = g \;\&\; \Big\{h \mid g = h \;\&\; [\![\exists x(Cx \wedge Ojx)]\!]^h_M = \emptyset\Big\} \cup \{h \mid g = h \;\&\; h(x) \in \mathcal{I}(P)\} \neq \emptyset\Big\}$$

$$= \begin{cases} \{f\} & \text{if } [\![\exists x(Cx \wedge Ojx)]\!]^f_M = \emptyset \text{ or } f(x) \in \mathcal{I}(P) \\ \emptyset & \text{otherwise} \end{cases}$$

- Note that in PL, (8) is equivalent to both (9) and (10).

(9) $\neg\exists x(Cx \wedge Ojx) \vee (\exists x(Cx \wedge Ojx) \wedge Px)$

(10) $\neg\exists x(Cx \wedge Ojx) \vee (\neg\neg\exists x(Cx \wedge Ojx) \wedge Px)$

- In DPL (8) is equivalent[4] to (10) but not (9); and (9) *would* capture the intended dependency when interpreted in DPL.
- So, apparently, we again have a situation where the PL equivalence based on double negation would be desirable.

[4] Equivalences in DPL:

Equivalence
$\phi \simeq \psi \Leftrightarrow$ for all M and f, $[\![\phi]\!]^f_M = [\![\psi]\!]^f_M$

Satisfaction-equivalence
$\phi \simeq_s \psi \Leftrightarrow$ for all M and f, $[\![\phi]\!]^f_M = \emptyset$ just in case $[\![\psi]\!]^f_M = \emptyset$

Uniqueness

However, it seems that we don't want ϕ to be *exactly* equivalent to $\neg\neg\phi$.

(11) ??It's not true that John doesn't own a shirt. It's in the wardrobe.

(12) ??Either John doesn't own a shirt, or it's in the wardrobe.

- Examples (11) and (12) sound strange in a way that their counterparts (5) and (7) respectively don't.
- The reason seems to be that these examples carry the implication[5] that, if John owns a car/shirt, then he owns exactly one—which is a much more plausible assumption in the case of cars than in the case of shirts.
- Here are some more contrasts:

[5] Let's leave open for the moment the question of what exactly this 'implication' amounts to.

(13) John owns a car. It's parked in a weird place. He owns another one which is in the garage.

(14) ??It's not true that John doesn't own a car. It's just parked in a weird place. He owns another one which is in the garage.

(15) ??Either John doesn't own a car, or it's parked in a weird place and he owns another one which is in the garage.

Matt Mandelkern (p.c.) has expressed doubts about uniqueness implications, on the basis of examples like (16).

(16) ?Either Sue didn't have a drink last night, or she had a second drink right after it.

- Thoughts? Personally I find this example strange, but admittedly have an interest in doing so.
- In what follows I'll present two accounts, with and without uniqueness implications.

Double Negation and Excluded Middle

- The non-equivalence of ϕ and $\neg\neg\phi$ in DPL is reminiscent of the situation in intuitionistic logic (IL).
- The parallel is by no means exact, since in IL this non-equivalence can be expressed as $\phi \not\dashv\vdash \neg\neg\phi$, whereas in DPL it can't really be brought out directly in terms of entailment or derivability.
- Nevertheless, it's worth looking at what one needs to add to IL in order to get the equivalence back. Famously, adding any of (17)–(19) to IL gets you classical logic.[6]

(17) $\neg\neg\phi \vdash \phi$ (double negation elimination)

(18) $\dfrac{\Gamma, \neg\phi \vdash \bot}{\Gamma \vdash \phi}$ (reductio ad absurdum)

(19) $\vdash \phi \vee \neg\phi$ (excluded middle)

- Question: could there be a way to achieve (something like) the double negation property for dynamic semantics by adding (something like) excluded middle? And could that help to resolve the issues we've identified with pronoun accessibility?
- Answer: *yes*, but it doesn't involve the standard DPL disjunction. Rather, it involves 'program disjunction',[7] defined in (20).

(20) $[\![\phi \cup \psi]\!]^f_M = [\![\phi]\!]^f_M \cup [\![\psi]\!]^f_M$

Like $\vee$, $\cup$ is internally static, but unlike $\vee$ it is externally dynamic. External dynamicity is crucial to the equivalence shown in Fact 1.[8]

Fact 1 *If* $\phi \simeq \phi \wedge \phi$ *then* $(\phi \cup \neg\phi) \wedge \neg\neg\phi \simeq \phi$

- In DPL $\phi \cup \neg\phi$ is a tautology,[9] but there are many semantically distinct tautologies in DPL. Consequently, DPL does not have the property that ϕ is equivalent to $T \wedge \phi$ for any DPL tautology T and formula ϕ. So much can be seen from Fact 1.
- Relevance for us: indefinites made inaccessible by double negation can be made accessible again on the assumption that the discourse is interpreted in the context of a specific tautology, namely, an appropriate instances of excluded middle with $\cup$.

[6] So e.g. in IL $(\phi \vee \neg\phi) \wedge \neg\neg\phi \dashv\vdash \phi$.

The connection is deeper than there is space to get into here. See Ranta [1994, 74–75] and Fernando [2001].

[7] [Groenendijk and Stokhof, 1991, 88]

[8] Groenendijk and Stokhof [1991, 63–64] discuss the conditions under which $\wedge$ is idempotent. In this paper we're concerned about the case where $\phi := \exists x(Cx \wedge Ojx)$, and in that case $\phi \simeq \phi \wedge \phi$.

Proof sketch: Groenendijk and Stokhof [1991] show that $(\phi \cup \psi) \wedge \chi \simeq (\phi \wedge \chi) \cup (\psi \wedge \chi)$. Therefore, $(\phi \cup \neg\phi) \wedge \neg\neg\phi \simeq (\phi \wedge \neg\neg\phi) \cup (\neg\phi \wedge \neg\neg\phi)$. If $\phi \simeq \phi \wedge \phi$ then $\phi \simeq \phi \wedge \neg\neg\phi$ and $[\![\neg\phi \wedge \neg\neg\phi]\!]^f_M = \emptyset$.

[9] I.e. for any M and f, $[\![\phi \cup \neg\phi]\!]^f_M \neq \emptyset$.

- For example, assume that (5) and (7) are interpreted in the context of $\exists x(Cx \wedge Ojx) \cup \neg\exists x(Cx \wedge Ojx)$. Then we have (21) and (22) respectively.

(21) $(\exists x(Cx \wedge Ojx) \cup \neg\exists x(Cx \wedge Ojx)) \wedge (\neg\neg\exists x(Cx \wedge Ojx) \wedge Px)$

$$\left[\!\left[(21)\right]\!\right]^f_M = \left\{ g \mid \begin{array}{l} g \in [\![\exists x(Cx \wedge Ojx)]\!]^f_M \cup [\![\neg\exists x(Cx \wedge Ojx)]\!]^f_M \text{ \& } [\![\exists x(Cx \wedge Ojx)]\!]^g_M \neq \emptyset \\ \text{\& } g(x) \in \mathcal{I}(P) \end{array} \right\}$$
$$= \left\{ g \mid g \in [\![\exists x(Cx \wedge Ojx)]\!]^f_M \text{ \& } g(x) \in \mathcal{I}(P) \right\}$$

So (2) $\simeq$ (21).

(22) $(\exists x(Cx \wedge Ojx) \cup \neg\exists x(Cx \wedge Ojx)) \wedge (\neg\exists x(Cx \wedge Ojx) \vee Px)$

$$\left[\!\left[(22)\right]\!\right]^f_M = \left\{ h \mid \begin{array}{l} \text{there's a } g : g \in [\![\exists x(Cx \wedge Ojx)]\!]^f_M \cup [\![\neg\exists x(Cx \wedge Ojx)]\!]^f_M \\ \text{\& } h \in [\![\neg\exists x(Cx \wedge Ojx) \vee Px]\!]^g_M \end{array} \right\}$$
$$= \left\{ g \mid (f = g \text{ \& } [\![\exists x(Cx \wedge Ojx)]\!]^g_M = \emptyset) \text{ or } \left(g \in [\![\exists x(Cx \wedge Ojx)]\!]^f_M \text{ \& } g(x) \in \mathcal{I}(P) \right) \right\}$$

So (9) $\simeq_s$ (22).

Note that in simple positive cases like (2) and single-negation cases like (4), adding the instance of excluded middle doesn't change anything, thanks to Facts 2 and 3.

Proof sketches: in DPL conjunction is associative, so we just have to show that $(\phi \cup \neg\phi) \wedge \phi \simeq \phi$ (Fact 2) and $(\phi \cup \neg\phi) \wedge \neg\phi \simeq \neg\phi$ (Fact 3). That's equivalent to showing that $(\phi \wedge \phi) \cup (\neg\phi \wedge \phi) \simeq \phi$ (Fact 2) and $(\phi \wedge \neg\phi) \cup (\neg\phi \wedge \neg\phi) \simeq \neg\phi$ (Fact 3). If $\phi \simeq \phi \wedge \phi$, this follows trivially.

Fact 2 *If* $\phi \simeq \phi \wedge \phi$ *then* $(\phi \cup \neg\phi) \wedge (\phi \wedge \psi) \simeq \phi \wedge \psi$

Fact 3 *If* $\phi \simeq \phi \wedge \phi$ *then* $(\phi \cup \neg\phi) \wedge (\neg\phi \wedge \psi) \simeq \neg\phi \wedge \psi$

TWO THINGS to note about this treatment of disjunction:

1. Binding is predicted to be symmetric, i.e. *either it's parked in a weird place, or John doesn't own a car* is predicted to be just as good as (7).
2. In either case the interpretation amounts to 'either John doesn't own a car, or some car he owns is parked in a weird place'. This take on the truth conditions of (7) is disputed by Krahmer and Muskens [1995]: for them, it should mean 'every car John owns is parked in a weird place'.[10]

[10] This is reminiscent of the difference between strong and weak readings of donkey sentences.

Both of these bugs/features follow from the (independently-given) semantics of $\vee$ in DPL. Both would be changed on the assumption that *p or q* is translated into DPL not as $p \vee q$ but as $\neg p \rightarrow q$. Nevertheless, the strong vs. weak issue at least is somewhat moot given the uniqueness effect, to be discussed next.

Uniqueness

Let's reflect on what program disjunction does in cases of an existential statement and its negation.

$$[\![\exists x Px \cup \neg\exists x Px]\!]^f_M = \begin{cases} \text{the set of } x\text{-variants of } f \text{ mapping } x \text{ to a } P, \text{ if there are any} \\ \{f\} \text{ otherwise} \end{cases}$$

If we want the anaphoric dependency to be passed on only in the case of uniqueness, then the input context for our unaugmented formulae should look like this instead:

$$\begin{cases} \text{the (singleton) set of } x\text{-variants of } f \text{ mapping } x \text{ to a } P\text{, if there's exactly one} \\ \{f\} \text{ otherwise} \end{cases}$$

That effect can be achieved by the introduction of an operator $\rceil$, defined in (23).

(23) $$[\![\rceil\phi]\!]^f_M = \begin{cases} [\![\phi]\!]^f_M & \text{if } \left|[\![\phi]\!]^f_M\right| = 1 \\ \{f\} & \text{otherwise} \end{cases}$$

Or, equivalently,

$$[\![\rceil\phi]\!]^f_M = \left\{g \mid g \in [\![\phi]\!]^f_M \;\&\; \left|[\![\phi]\!]^f_M\right| = 1\right\} \cup \left\{g \mid f = g \;\&\; \left|[\![\phi]\!]^g_M\right| \neq 1\right\}$$

- Note that $\rceil\phi$ is also a DPL tautology (for any ϕ). I will henceforth refer to formulae of the form $\rceil\phi$ as instances of 'unique excluded middle' (UEM).[11]
- If we now use UEM for our augmentations of (6) and (8) instead of excluded middle with $\cup$, we get the uniqueness effect, as seen below in (24) and (25) respectively.

[11] In a previous version of this material, UEM was defined as a formula of the form $\rceil(\phi \cup \neg\phi)$. That was before I realised that this is equivalent to $\rceil\phi$.

(24) $\rceil\exists x(Cx \wedge Ojx) \wedge (\neg\neg\exists x(Cx \wedge Ojx) \wedge Px)$

$$\left[\!\!\left[(24)\right]\!\!\right]^f_M = \left\{ g \;\middle|\; \begin{array}{l} g \in \left\{h \mid h \in [\![\exists x(Cx \wedge Ojx)]\!]^f_M \;\&\; \left|[\![\exists x(Cx \wedge Ojx)]\!]^f_M\right| = 1\right\} \\ \quad \cup \left\{h \mid f = h \;\&\; \left|[\![\exists x(Cx \wedge Ojx)]\!]^h_M\right| \neq 1\right\} \\ \& \; [\![\exists x(Cx \wedge Ojx)]\!]^f_M \neq \emptyset \;\&\; g(x) \in \mathcal{I}(P) \end{array} \right\}$$

$$= \left\{ g \;\middle|\; \begin{array}{l} \left([\![\exists x(Cx \wedge Ojx)]\!]^f_M = \{g\} \;\&\; g(x) \in \mathcal{I}(P)\right) \\ \text{or } \left(f = g \;\&\; \left|[\![\exists x(Cx \wedge Ojx)]\!]^f_M\right| > 1 \;\&\; g(x) \in \mathcal{I}(P)\right) \end{array} \right\}$$

'Either John owns exactly one car, which is parked in a weird place, or John owns more than one car and x is parked in a weird place' (with x free).

(25) $\rceil\exists x(Cx \wedge Ojx) \wedge (\neg\exists x(Cx \wedge Ojx) \vee Px)$

$$\left[\!\!\left[(25)\right]\!\!\right]^f_M = \left\{ g \;\middle|\; \begin{array}{l} g \in \left\{h \mid h \in [\![\exists x(Cx \wedge Ojx)]\!]^f_M \;\&\; \left|[\![\exists x(Cx \wedge Ojx)]\!]^f_M\right| = 1\right\} \\ \quad \cup \left\{h \mid f = h \;\&\; \left|[\![\exists x(Cx \wedge Ojx)]\!]^h_M\right| \neq 1\right\} \\ \& \; [\![\neg\exists x(Cx \wedge Ojx)]\!]^f_M \cup [\![Px]\!]^f_M \neq \emptyset \end{array} \right\}$$

$$= \left\{ g \;\middle|\; \begin{array}{l} \left([\![\exists x(Cx \wedge Ojx)]\!]^f_M = \{g\} \;\&\; g(x) \in \mathcal{I}(P)\right) \\ \text{or } \left(f = g \;\&\; [\![\exists x(Cx \wedge Ojx)]\!]^g_M = \emptyset\right) \\ \text{or } \left(f = g \;\&\; \left|[\![\exists x(Cx \wedge Ojx)]\!]^f_M\right| > 1 \;\&\; g(x) \in \mathcal{I}(P)\right) \end{array} \right\}$$

'Either John doesn't own a car, or he owns exactly one car, which is parked in a weird place, or he owns more than one car and x is parked in a weird place' (with x free).

- As noted above, this interpretation abolishes the distinction between weak and strong readings by making *a car* inaccessible as an antecedent to *it* if John owns more than one car.

- As before, adding UEM to a simple positive formula like (2) doesn't change anything (Fact 4). Adding UEM to a single-negated sentence like (4) doesn't change anything either (Fact 5).

Fact 4 *If* $\phi \simeq \phi \wedge \phi$ *then* $\rceil\phi \wedge (\phi \wedge \psi) \simeq \phi \wedge \psi$

Fact 5 *If* $\phi \simeq \phi \wedge \phi$ *then* $\rceil\phi \wedge (\neg\phi \wedge \psi) \simeq \neg\phi \wedge \psi$

Proof sketches: If $\phi \simeq \phi \wedge \phi$ then $[\![\rceil\phi \wedge (\phi \wedge \psi)]\!]^f_M =$

$$\begin{cases} [\![\phi \wedge (\phi \wedge \psi)]\!]^f_M = [\![(\phi \wedge \phi) \wedge \psi]\!]^f_M \\ = [\![\phi \wedge \psi]\!]^f_M \text{ if } \left|[\![\phi]\!]^f_M\right| = 1 \\ [\![\phi \wedge \psi]\!]^f_M \text{ otherwise} \end{cases}$$

$= [\![\phi \wedge \psi]\!]^f_M$, and $[\![\rceil\phi \wedge (\neg\phi \wedge \psi)]\!]^f_M =$

$$\begin{cases} [\![\phi \wedge (\neg\phi \wedge \psi)]\!]^f_M = [\![(\neg\phi \wedge \phi) \wedge \psi]\!]^f_M \\ = \emptyset \text{ if } \left|[\![\phi]\!]^f_M\right| = 1 \\ [\![\neg\phi \wedge \psi]\!]^f_M \text{ otherwise} \end{cases}$$

$= [\![\neg\phi \wedge \psi]\!]^f_M$.

Composition

- Obvious question: where do instances of EM/UEM come from?
- One thought: treat them as introduced lexically by negation as a kind of projective content. That is to say, in addition to introducing $\neg\phi$ in the standard dimension of meaning, instances of negation introduce $\phi \cup \neg\phi$ or $\rceil\phi$ in another dimension of meaning.

An alternative: they're introduced by a weird kind of accommodation.

- Doing this properly requires moving from DPL to a dynamic semantic system that permits compositionality below the level of the clause,[12] so I'll just give a schematic treatment in Figure 2 (assuming UEM, and with apologies to Potts [2005]).

[12] In the full version of this paper [Gotham, 2019, §5] I use CDRT [Muskens, 1996].

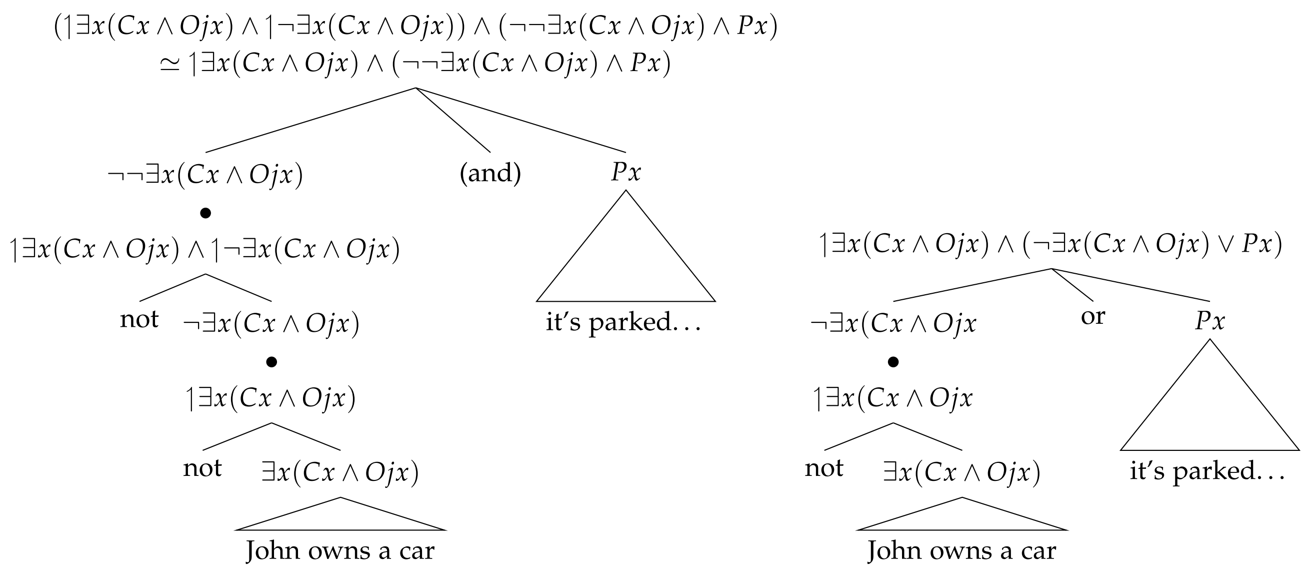

Figure 2: Compositional (sort of) interpretations of (5) and (7)

Discussion

- On the one hand, the approach in this paper is very conservative about the underlying logic of dynamics.[13] I haven't changed the clauses for any existing DPL connective.

[13] I've used DPL but it would work just as well with most any other theory.

- In particular, I haven't actually made $\vee$ internally dynamic or $\neg$ involutive. In case it isn't obvious yet, the binding in these cases is actually that implied by the underlining below.

  $(\underline{\exists x} Fx \cup \neg\exists x Fx) \wedge (\neg\neg\exists x Fx \wedge G\underline{x})$
  $(\underline{\exists x} Fx \cup \neg\exists x Fx) \wedge (\neg\exists x Fx \vee G\underline{x})$

- On the other hand, it requires a novel compositional (or other) story of how these instances of EM/UEM are introduced.

- There's a ready account of the uniqueness effect ... if that effect is real.

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