Double negation, excluded middle and accessibility in dynamic semantics

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1 Introduction

As is well known...

- In pretty much all dynamic semantic theories, negating a formula (DRS, file...) results in a **test**: a formula that doesn't introduce any new anaphoric dependencies that can be picked up in subsequent discourse.
- It follows that indefinites introduced in the scope of negation are inaccessible as antecedents to pronouns outside the scope of negation.
- In the simplest cases, this is a welcome result. However, the fact that this 'test-making' is irreversible can lead to unwelcome results in case of double negation and (possibly) related examples.

In this paper, I outline a way of making inaccessible indefinites accessible again, inspired by intuitionistic logic.

2 Puzzles of accessibility

First-generation dynamic semantic theories[1] were introduced in part to handle discourses such as (1), in which a pronoun is (apparently) outside the scope of its binder.

(1) John owns a^x car. It$_x$ is parked outside.

The standard move is to treat a sentence meaning as some kind of transition between variable assignments or related structures. As an example, see the semantics of DPL (Groenendijk & Stokhof 1991: 54), which has the same syntax as standard predicate logic (PL).

[1] E.g. File Change Semantics (FCS, Heim 1982, 1983), Dynamic Predicate Logic (DPL, Groenendijk & Stokhof 1991), Discourse Representation Theory (DRT, Kamp 1981, Kamp & Reyle 1993, Kamp, van Genabith & Reyle 2011), Compositional DRT (CDRT, Muskens 1996), Incremental Dynamics (ID, van Eijck 2001, Nouwen 2003).

Definition 1 (Semantics of DPL)

$$[\![Pt_1 \dots t_n]\!]^{M,g} = \left\{h \mid g = h \textit{ and } \left\langle [\![t_1]\!]^{M,h}, \dots, [\![t_n]\!]^{M,h} \right\rangle \in \mathcal{I}(P)\right\}$$
$$[\![t_1 = t_2]\!]^{M,g} = \left\{h \mid g = h \textit{ and } [\![t_1]\!]^{M,g} = [\![t_2]\!]^{M,g}\right\}$$
$$[\![\neg\phi]\!]^{M,g} = \left\{h \mid g = h \textit{ and } [\![\phi]\!]^{M,h} = \emptyset\right\}$$
$$[\![\phi \wedge \psi]\!]^{M,g} = \left\{h \mid \textit{there's a } k : k \in [\![\phi]\!]^{M,g} \textit{ and } h \in [\![\psi]\!]^{M,k}\right\}$$
$$[\![\phi \vee \psi]\!]^{M,g} = \left\{h \mid g = h \textit{ and } [\![\phi]\!]^{M,h} \cup [\![\psi]\!]^{M,h} \neq \emptyset\right\}$$
$$[\![\phi \to \psi]\!]^{M,g} = \left\{h \mid g = h \textit{ and } [\![\phi]\!]^{M,h} \subseteq \left\{k \mid [\![\psi]\!]^{M,k} \neq \emptyset\right\}\right\}$$
$$[\![\exists x\phi]\!]^{M,g} = \left\{h \mid \textit{there's a } k : g[x]k \textit{ and } h \in [\![\phi]\!]^{M,k}\right\}$$
$$[\![\forall x\phi]\!]^{M,g} = \left\{h \mid g = h \textit{ and } \{k \mid h[x]k\} \subseteq \left\{k \mid [\![\phi]\!]^{M,k} \neq \emptyset\right\}\right\}$$

□

- Note that in the clauses for predication, negation, disjunction, implication and the universal quantifier, the set of output assignments for input assignment g is either $\{g\}$ or $\emptyset$. Formulae with this property are called **tests**.
- In DPL, (1) can be translated as shown in (2), in which the variable representing the pronoun really is outside the syntactic scope of the quantifier representing its binder. And yet, the bound effect is achieved, as shown in (3).

(2) $\exists x(Cx \wedge Ojx) \wedge Px$

(3)
$$\begin{aligned}[\![(2)]\!]^{M,g} &= \left\{h \mid \text{there's a } k : k \in [\![\exists x(Cx \wedge Ojx)]\!]^{M,g} \text{ and } h \in [\![Px]\!]^{M,k}\right\}\\
&= \left\{h \mid \text{there's a } k : k \in [\![\exists x(Cx \wedge Ojx)]\!]^{M,g} \text{ and } k = h \text{ and } h(x) \in \mathcal{I}(P)\right\}\\
&= \left\{h \mid h \in [\![\exists x(Cx \wedge Ojx)]\!]^{M,g} \text{ and } h(x) \in \mathcal{I}(P)\right\}\\
&= \Big\{h \mid \text{there's a } k : g[x]k \text{ and there's an } f : f \in [\![Cx]\!]^{M,k}\\
&\qquad \text{and } h \in [\![Ojx]\!]^{M,f} \text{ and } h(x) \in \mathcal{I}(P)\Big\}\\
&= \{h \mid g[x]h \text{ and } h(x) \in \mathcal{I}(C) \text{ and } \langle \mathcal{I}(j), h(x)\rangle \in \mathcal{I}(O) \text{ and } h(x) \in \mathcal{I}(P)\}\end{aligned}$$

In contrast, no binding is possible in the case of (4), the translation of which is given in (5) and interpreted as shown in (6).

(4) John does not own a^{x} car. It$_{\#x}$ is parked outside.

(5) $\neg\exists x(Cx \wedge Ojx) \wedge Px$

(6)
$$\begin{aligned}[\![(5)]\!]^{M,g} &= \left\{h \mid g = h \text{ and } [\![\exists x(Cx \wedge Ojx)]\!]^{M,h} = \emptyset \text{ and } h(x) \in \mathcal{I}(P)\right\}\\
&= \left\{h \mid g = h \text{ and } \left\{k \mid h[x]k \text{ and } k \in [\![Cx \wedge Ojx]\!]^{M,h}\right\} = \emptyset \text{ and } h(x) \in \mathcal{I}(P)\right\}\end{aligned}$$

2.1 Negation

In the case of (4), this is a welcome result.[2] However, the effect persists in the case of (7), where it is less welcome.

(7) It's not the case that John does not own a^x car. It$_x$ is parked outside.

(7) is translated into DPL as shown in (8), which is interpreted as shown in (9).

(8) $\neg\neg\exists x(Cx \wedge Ojx) \wedge Px$

(9)
$$\begin{aligned}[\![(8)]\!]^{M,g} &= \Big\{h \mid g = h \text{ and } [\![\exists x(Cx \wedge Ojx)]\!]^{M,h} \neq \emptyset \text{ and } h(x) \in \mathcal{I}(P)\Big\} \\ &= \Big\{h \mid g = h \text{ and } \Big\{k \mid h[x]k \text{ and } k \in [\![Cx \wedge Ojx]\!]^{M,h}\Big\} \neq \emptyset \text{ and } h(x) \in \mathcal{I}(P)\Big\}\end{aligned}$$

All the frameworks mentioned in footnote 1 are the same in this respect. The issue is that, in general, ϕ and $\neg\neg\phi$ are not equivalent in these frameworks—unlike in PL. As Groenendijk & Stokhof (1991: 62) put it:

> [W]e may conclude that the law of double negation will not hold unconditionally. Consider a formula ϕ that is not a test. Negating ϕ results in the test $\neg\phi$, and a second negation, which gives $\neg\neg\phi$, does not reverse this effect [...] However, as far as their truth conditions are concerned, the two coincide

Which brings us to the following definitions (Groenendijk & Stokhof 1991: 55–56):

Definition 2 (Truth in DPL)

- *ϕ is **true$_g$ in M** $\Leftrightarrow$ $[\![\phi]\!]^{M,g} \neq \emptyset$.*
- *ϕ is **true in M** $\Leftrightarrow$ for every g, ϕ is true$_g$ in M.*

□

Definition 3 (Equivalences in DPL)

- *ϕ is **equivalent** to ψ (written $\phi \simeq \psi$) $\Leftrightarrow$ for all M and g, $[\![\phi]\!]^{M,g} = [\![\psi]\!]^{M,g}$.*
- *ϕ is **satisfaction-equivalent** to ψ (written $\phi \simeq_s \psi$) $\Leftrightarrow$ for all M, ϕ is true in M iff ψ is true in M.*

□

The issue, then, is that

$$\exists x(Cx \wedge Ojx) \simeq_s \neg\neg\exists x(Cx \wedge Ojx)$$

but

$$\exists x(Cx \wedge Ojx) \not\simeq \neg\neg\exists x(Cx \wedge Ojx)$$

[2] As the # indicates.

2.2 Disjunction

The same issue seems to be implicated in another puzzle for pronoun accessibility in dynamic semantics, illustrated by (10), which is most naturally translated into DPL as shown in (11).

(10) Either John doesn't own^x a car, or it$_x$ is parked outside.

(11) $\neg\exists x(Cx \wedge Ojx) \vee Px$

As the interpretation shown in (12) shows, this DPL translation does not make the indefinite accessible as an antecedent to the pronoun.

(12) $[\![(11)]\!]^{M,g} = \Big\{h \mid g = h \text{ and } [\![\neg\exists x(Cx \wedge Ojx)]\!]^{M,h} \cup [\![Px]\!]^{M,h} \neq \emptyset\Big\}$

In PL, (11) is equivalent to (13) and (14).

(13) $\neg\exists x(Cx \wedge Ojx) \vee (\exists x(Cx \wedge Ojx) \wedge Px)$

(14) $\neg\exists x(Cx \wedge Ojx) \vee (\neg\neg\exists x(Cx \wedge Ojx) \wedge Px)$

In DPL, (11) is equivalent to (14) but not (13). And (13) *would* capture the binding relationship in (10) when interpreted in DPL. So apparently, we again have a situation in which the PL equivalence based on double negation would be desirable.

3 Excursus: intuitionistic logic

Another logic in which ϕ and $\neg\neg\phi$ are non-equivalent is intuitionistic logic (IL). In IL:

$$\phi \vdash \neg\neg\phi$$

but

$$\neg\neg\phi \nvdash \phi$$

Another principle of classical logic that intuitionistic logic lacks is **excluded middle** (EM). That is to say, in IL:

$$\nvdash \phi \vee \neg\phi$$

In a very strong sense, double negation elimination (DNE) and EM are equivalent commitments: adding either one as an axiom to IL results in classical logic. A hint of this can be seen from Proof 1.

Proof 1 (Adding EM as an axiom to IL validates DNE)

$$\dfrac{\dfrac{}{\vdash p \vee \neg p} \quad \dfrac{}{p \vdash p} \quad \dfrac{\dfrac{\dfrac{}{\neg\neg p \vdash \neg\neg p} \quad \dfrac{}{\neg p \vdash \neg p}}{\neg\neg p, \neg p \vdash \bot}\,\neg_E}{\neg\neg p, \neg p \vdash p}\,\text{EFSQ}}{\neg\neg p \vdash p}\,\vee_E$$

■

This invites the following thought: could there be a way to achieve (something like) the double negation property for dynamic semantics by adding (something like) excluded middle? And could that help to resolve the issues we've identified with pronoun accessibility?

4 Back to DPL

The parallel with IL is by no means exact; it can't be stated in terms of entailment, for instance, since in DPL $\neg\neg\phi$ *does* entail ϕ, given the definition of entailment shown in Definition 4 (Groenendijk & Stokhof 1991: 70).

Definition 4 (Entailment in DPL)

$\phi_1, \ldots, \phi_n \vDash \psi \Leftrightarrow$ *for all* $M, h, g_1, \ldots, g_n$, *if* $g_2 \in [\![\phi_1]\!]^{M,g_1}$ *and … and* $h \in [\![\phi_n]\!]^{M,g_n}$, *then* $[\![\psi]\!]^{M,h} \neq \emptyset$

□

However, the difference between $\neg\neg\phi$ and ϕ can still be usefully brought out in terms of DPL entailment, by noting that in DPL:

$$\exists x Fx \vDash Fx$$

but

$$\neg\neg\exists x Fx \nvDash Fx$$

- In a limited sense, then, DPL-entailment is a test for accessibility.
- The question then becomes, is there a form of excluded middle we could add that would have something like the effect of EM in Proof 1?
- The answer is *yes*, but it doesn't involve the standard DPL disjunction. Rather, it involves what Groenendijk & Stokhof (1991: 88) call 'program disjunction'.[3]

Definition 5 (Program disjunction) *Extend the language of DPL with the following clauses:*

- *If* ϕ *and* ψ *are formulae, then* $\phi \curlyvee \psi$ *is a formula*
- $[\![\phi \curlyvee \psi]\!]^{M,g} = [\![\phi]\!]^{M,g} \cup [\![\psi]\!]^{M,g}$

□

- Note that a formula formed by program disjunction is *not* necessarily a test (unlike standard disjunction).
- The connective is, in G&S's terms, **internally static** but **externally dynamic**.
- The external dynamicity is crucial in Proof 2, which makes use of a program disjunction form of EM.

Proof 2 ($\exists x Fx \curlyvee \neg\exists x Fx, \neg\neg\exists x Fx \vDash Fx$**)**

$$[\![\exists x Fx \curlyvee \neg\exists x Fx]\!]^{M,g} = \{h \mid g[x]h \text{ and } h(x) \in \mathcal{I}(F)\} \cup \left\{h \mid g = h \text{ and } [\![\exists x Fx]\!]^{M,h} = \emptyset\right\}$$

$$[\![\neg\neg\exists x Fx]\!]^{M,g} = \left\{h \mid g = h \text{ and } [\![\exists x Fx]\!]^{M,h} \neq \emptyset\right\}$$

$$[\![Fx]\!]^{M,g} = \{h \mid g = h \text{ and } h(x) \in \mathcal{I}(F)\}$$

[3] By analogy with disjunction in dynamic logics for computer programming languages. I'm using the notation from Groves 2000, though.

Case 1 *Suppose that* $[\![\exists xFx]\!]^{M,g} = \emptyset$. *Then* $[\![\exists xFx \curlyvee \neg\exists xFx]\!]^{M,g} = \emptyset \cup \{g\} = \{g\}$. *And then* $[\![\neg\neg\exists xFx]\!]^{M,g} = \emptyset$*, from which* Fx *follows trivially.*

Case 2 *Suppose that* $[\![\exists xFx]\!]^{M,g} \neq \emptyset$. *Then* $[\![\exists xFx \curlyvee \neg\exists xFx]\!]^{M,g} = \{h \mid g[x]h$ *and* $h(x) \in \mathcal{I}(F)\}$. *For any* h *in this set,* $[\![\neg\neg\exists xFx]\!]^{M,h} = \{h\}$*, and then* $[\![Fx]\!]^{M,h} = \{h\}$. *So* Fx *follows.*

■

- Importantly, for Proof 2 to go through, *both* premises are required: $\exists xFx \curlyvee \neg\exists xFx \not\models Fx$.
- Neither $\exists xFx \curlyvee \neg\exists xFx$ nor $\neg\neg\exists xFx$ makes x accessible, but their combination does.

Now we can have a look at the effect of adding EM to our previous examples.

5 Back to our examples

5.1 Double negation in DPL in the context of EM

Let's consider example (7) again, but this time let's add to the translation given in (8) an appropriate instance of EM, as shown in (15).

(7) It's not the case that John doesn't own a^x car. It$_x$ is parked outside.

(15) $(\exists x(Cx \wedge Ojx) \curlyvee \neg\exists x(Cx \wedge Ojx)) \wedge (\neg\neg\exists x(Cx \wedge Ojx) \wedge Px)$

(15) is interpreted as shown in (16).

(16)
$$\begin{aligned}
[\![(15)]\!]^{M,g} &= \Big\{h \mid \text{there's a } k \text{ and } f: k \in [\![\exists x(Cx \wedge Ojx) \curlyvee \neg\exists x(Cx \wedge Ojx)]\!]^{M,g} \\
&\qquad \text{and } f \in [\![\neg\neg\exists x(Cx \wedge Ojx)]\!]^{M,k} \text{ and } h \in [\![Px]\!]^{M,f}\Big\} \\
&= \Big\{h \mid h \in [\![\exists x(Cx \wedge Ojx)]\!]^{M,g} \cup [\![\neg\exists x(Cx \wedge Ojx)]\!]^{M,g} \\
&\qquad \text{and } [\![\exists x(Cx \wedge Ojx)]\!]^{M,h} \neq \emptyset \text{ and } h(x) \in \mathcal{I}(P)\Big\} \\
&= \Big\{h \mid h \in \{h \mid g[x]h \text{ and } h(x) \in \mathcal{I}(C) \text{ and } \langle \mathcal{I}(j), h(x)\rangle \in \mathcal{I}(O)\} \\
&\qquad\qquad \cup \Big\{h \mid g = h \text{ and } [\![\exists x(Cx \wedge Ojx)]\!]^{M,h} = \emptyset\Big\} \\
&\qquad \text{and } [\![\exists x(Cx \wedge Ojx)]\!]^{M,h} \neq \emptyset \text{ and } h(x) \in \mathcal{I}(P)\Big\} \\
&= \{h \mid g[x]h \text{ and } h(x) \in \mathcal{I}(C) \text{ and } \langle \mathcal{I}(j), h(x)\rangle \in \mathcal{I}(O) \text{ and } h(x) \in \mathcal{I}(P)\}
\end{aligned}$$

The last line of reasoning is justified by the inference that $\Big\{h \mid g = h \text{ and } [\![\exists x(Cx \wedge Ojx)]\!]^{M,h} = \emptyset\Big\} = \emptyset$, since we know that $[\![\exists x(Cx \wedge Ojx)]\!]^{M,h} \neq \emptyset$. After that, the clause $[\![\exists x(Cx \wedge Ojx)]\!]^{M,h} \neq \emptyset$ is redundant.

5.2 Two forms of disjunction

Now let's consider example (10) again. In (17) we add the appropriate instance of EM to the DPL translation given in (11). Interpretation then proceeds as shown in (18).

(10) Either John doesn't own^x a car, or it$_x$ is parked outside.

(17) $(\exists x(Cx \wedge Ojx) \curlyvee \neg\exists x(Cx \wedge Ojx)) \wedge (\neg\exists x(Cx \wedge Ojx) \vee Px)$

(18)
$$[\![(17)]\!]^{M,g} = \Big\{ h \mid h \in \{h \mid g[x]h \text{ and } h(x) \in \mathcal{I}(C) \text{ and } \langle \mathcal{I}(j), h(x)\rangle \in \mathcal{I}(O)\}$$
$$\cup \Big\{ h \mid g = h \text{ and } [\![\exists x(Cx \wedge Ojx)]\!]^{M,h} = \emptyset \Big\}$$
$$\text{and } \Big\{ k \mid h = k \text{ and } [\![\exists x(Cx \wedge Ojx)]\!]^{M,k} = \emptyset \Big\}$$
$$\cup \{k \mid h = k \text{ and } k(x) \in \mathcal{I}(P)\} \neq \emptyset \Big\}$$

To evaluate (18) we'll consider **three cases**:

1. John doesn't own a car, in which case (17) should be true.
2. John owns a car which is parked outside, in which case (17) should be true.
3. John owns a car but no car he owns is parked outside, in which case (17) should be false.[4]

In **case 1**, $\{h \mid g[x]h \text{ and } h(x) \in \mathcal{I}(C) \text{ and } \langle \mathcal{I}(j), h(x)\rangle \in \mathcal{I}(O)\} = \emptyset$, and so we can disregard $\{k \mid h = k \text{ and } k(x) \in \mathcal{I}(P)\}$ and reduce (18) to (19).

(19)
$$\Big\{ h \mid g = h \text{ and } [\![\exists x(Cx \wedge Ojx)]\!]^{M,h} = \emptyset \text{ and } \Big\{ k \mid h = k \text{ and } [\![\exists x(Cx \wedge Ojx)]\!]^{M,k} = \emptyset \Big\} \neq \emptyset \Big\}$$
$$= \Big\{ h \mid g = h \text{ and } [\![\exists x(Cx \wedge Ojx)]\!]^{M,h} = \emptyset \text{ and } [\![\exists x(Cx \wedge Ojx)]\!]^{M,h} = \emptyset \Big\}$$
$$= \Big\{ h \mid g = h \text{ and } [\![\exists x(Cx \wedge Ojx)]\!]^{M,h} = \emptyset \Big\}$$

By hypothesis, John doesn't own a car, and so (19) is non-empty and (17) is true.

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In **case 2 or 3**, $[\![\exists x(Cx \wedge Ojx)]\!]^{M,g} \neq \emptyset$ (for any g), and so (18) reduces to (20).

(20) $\{h \mid g[x]h \text{ and } h(x) \in \mathcal{I}(C) \text{ and } \langle \mathcal{I}(j), h(x)\rangle \in \mathcal{I}(O) \text{ and } h(x) \in \mathcal{I}(P)\}$

By hypothesis, John owns a car, and so (20) is empty, and hence (17) is false, just in case no car he owns is parked outside.

■

[4] Krahmer & Muskens (1995) dispute this take on the truth conditions of (10). For them, it requires, if John owns a car, that *all* of John's cars are parked outside. The difference in interpretation here is highly reminiscent of the difference between weak and strong readings of donkey sentences. We can get the strong reading on the assumption (which Krahmer & Muskens make) that *p or q* is translated not as $p \vee q$ but as $\neg p \to q$. The reader is invited to check that $(\exists x(Cx \wedge Ojx) \curlyvee \neg\exists x(Cx \wedge Ojx)) \wedge (\neg\neg\exists x(Cx \wedge Ojx) \to Px)$ gets the right interpretation in this case.

6 Prospects

So far so good, but this discussion raises (at least) two major questions:

1. Where would these 'appropriate instances of EM' come from?
2. Is $\curlyvee$ a legitimate connective?

We'll take these questions in turn, beginning with some wild speculation.

6.1 Excluded middle as conventional implicature?

Kubota & Uegaki (2009) present a theory of conventional implicatures (CIs) according to which

- individual lexical items can contribute both at-issue content and CIs (which are not at-issue), and
- CIs take scope.

The theory is couched in terms of a continuized grammar (Barker & Shan 2014), but can be reconstituted in terms of covert movement for sake of exposition. Take (21).

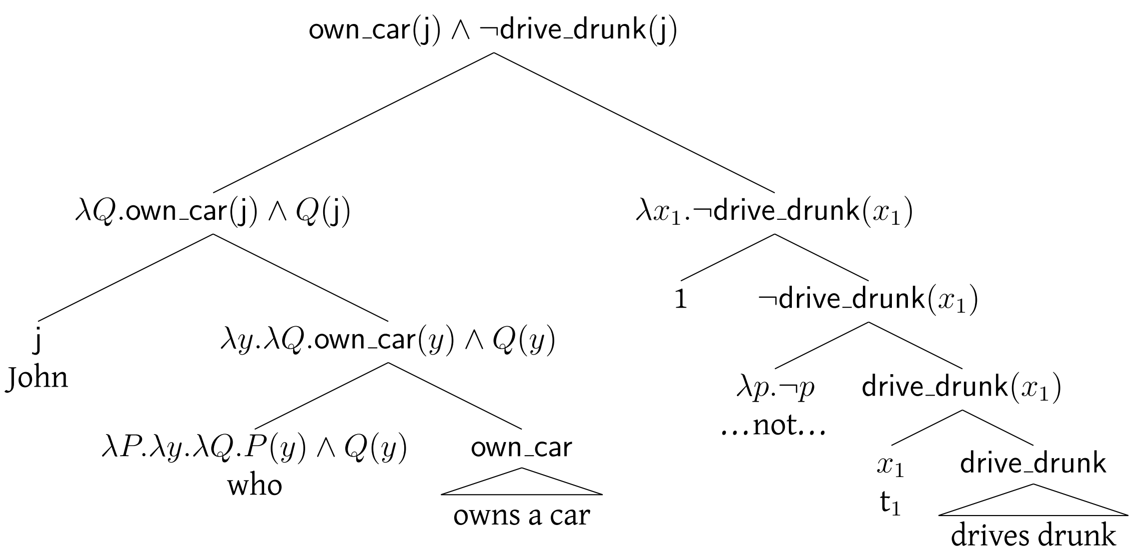
(21) It's not true that John, who owns a car, drives drunk.

(21) is interpreted as asserting:

- John owns a car.
- John does not drive drunk.

In other words, despite their surface positions, only John's drunk-driving is semantically in the scope of negation. His car ownership, as a CI, is not denied. Kubota & Uegaki's theory of what's going on here can be represented by the LF shown in (22).

(22)



The CI trigger *who* takes its arguments, and then the structure so built moves to a scope-taking position.

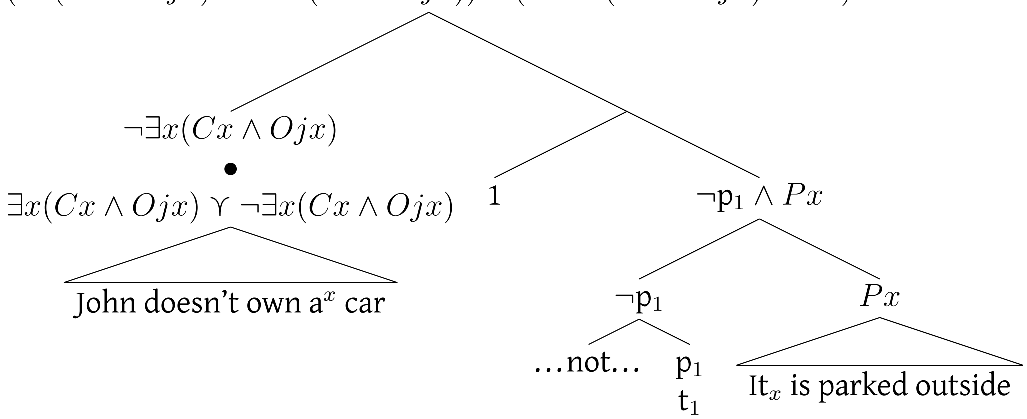
6.1.1 What about EM?

Let's run with the thought that

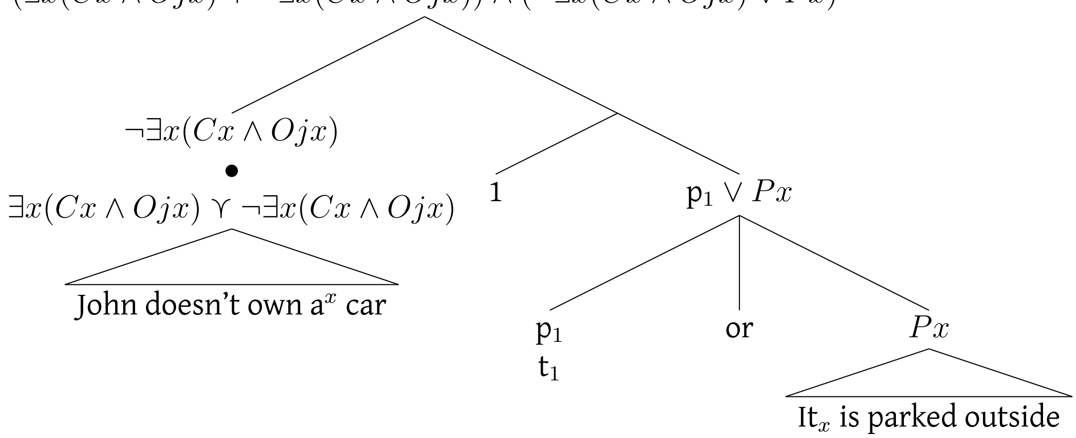
- *negation* could be CI trigger,
- the CI triggered by the negation of p is $p \curlyvee \neg p$: the appropriate instance of EM.

We can then imagine the LF of (7) is something like (23), and the the LF of (10) is something like (24).

(23) $(\exists x(Cx \wedge Ojx) \curlyvee \neg\exists x(Cx \wedge Ojx)) \wedge (\neg\neg\exists x(Cx \wedge Ojx) \wedge Px)$



(24) $(\exists x(Cx \wedge Ojx) \curlyvee \neg\exists x(Cx \wedge Ojx)) \wedge (\neg\exists x(Cx \wedge Ojx) \vee Px)$



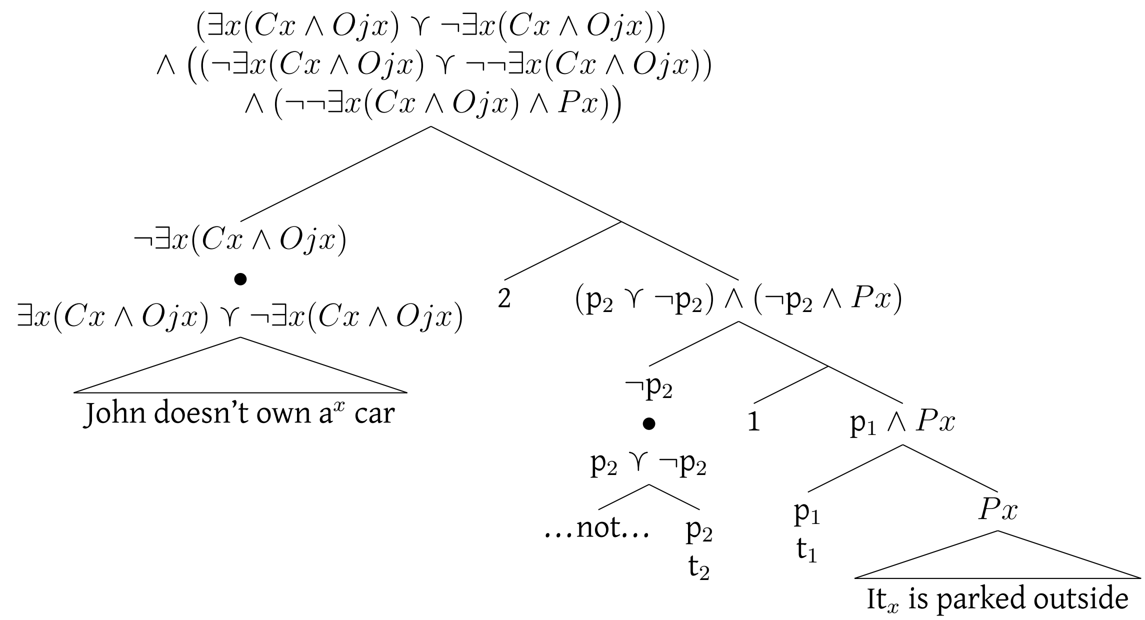
- The bullet notation, separating at-issue from not-at-issue content, is borrowed from Potts (2005) and doesn't represent an actual analysis at this stage.
- In fact, to get compositional semantics below the level of the sentence we need to move beyond DPL.
- That'll take us out of reach of this paper, but it should be clear that it can be done.

The presentation of (23), though, raises another question: what about the second negation?

6.1.2 Double negation and CIs

Given the suggestion that negation is a CI trigger, there should be two CI triggers in the case of double negation. So really, (23) should look like (25).

(25)



In the final interpretation, the extra conjunct doesn't change anything, since $[\![\neg\exists x(Cx \wedge Ojx) \curlyvee \neg\neg\exists x(Cx \wedge Ojx)]\!]^{M,g} = \{g\}$ (for any assignment g).

6.2 The status of program disjunction

- One way of bringing out the worry implicit in this question is by thinking about what would happen if we tried to formulate a version of $\curlyvee$ in a dynamic semantic framework that makes use of partial (rather than total) variable assignments, such as DRT, FCS or (in a sense) ID.
- If $[\![\phi]\!]^{M,g}$ and $[\![\psi]\!]^{M,g}$ are sets of partial assignments, and $[\![\phi \curlyvee \psi]\!]^{M,g} = [\![\phi]\!]^{M,g} \cup [\![\psi]\!]^{M,g}$, then you could have $g, h \in [\![\phi \curlyvee \psi]\!]^{M,g}$ such that g and h have different domains.
- In fact, precisely this *will* happen in the kind of case we've been looking at, where $\phi := \exists x Fx$ and $\psi := \neg\exists x Fx$.
- So for example, in FCS there would be no sensible answer to the question of what dom($\mathtt{F} + (\phi \curlyvee \psi)$) is: $\mathtt{F}+(\phi \curlyvee \psi)$ wouldn't be a file.
- One can always answer 'so much the worse for partial assignments'. But maybe we can achieve the same effect without program disjunction?

We'll begin by proving a de Morgan-like equivalence.

Proof 3 ($(p \curlyvee q) \wedge r \simeq_s (p \wedge r) \vee (q \wedge r)$) *We proceed in two steps:*

1. $(p \curlyvee q) \wedge r \simeq (p \wedge r) \curlyvee (q \wedge r)$ *(Groenendijk & Stokhof 1991: 88)*

2. $(p \wedge r) \curlyvee (q \wedge r) \simeq_s (p \wedge r) \vee (q \wedge r)$

For step 2, we prove the more general statement $p \curlyvee q \simeq_s p \vee q$

$$\begin{aligned} p \curlyvee q \textit{ is true}_g &\Leftrightarrow [\![p \curlyvee q]\!]^{M,g} \neq \emptyset \\ &\Leftrightarrow [\![p]\!]^{M,g} \cup [\![q]\!]^{M,g} \neq \emptyset \end{aligned}$$

$$\begin{aligned} p \vee q \textit{ is true}_g &\Leftrightarrow [\![p \vee q]\!]^{M,g} \neq \emptyset \\ &\Leftrightarrow \Big\{h \mid g = h \textit{ and } [\![p]\!]^{M,h} \cup [\![q]\!]^{M,h} \neq \emptyset\Big\} \neq \emptyset \\ &\Leftrightarrow [\![p]\!]^{M,g} \cup [\![q]\!]^{M,g} \neq \emptyset \end{aligned}$$

■

Since (*ex hypothesi*) our cases of EM with program disjunction take widest possible scope, they can be systematically replaced with satisfaction-equivalent forms using only standard disjunction. We'll finish by demonstrating this for our two examples. From Proof 3,

$$(15) \simeq_s (26)$$

and

$$(17) \simeq_s (27)$$

(15) $(\exists x(Cx \wedge Ojx) \curlyvee \neg\exists x(Cx \wedge Ojx)) \wedge (\neg\neg\exists x(Cx \wedge Ojx) \wedge Px)$

(26) $\big(\exists x(Cx \wedge Ojx) \wedge (\neg\neg\exists x(Cx \wedge Ojx) \wedge Px)\big) \vee \big(\neg\exists x(Cx \wedge Ojx) \wedge (\neg\neg\exists x(Cx \wedge Ojx) \wedge Px)\big)$

(17) $(\exists x(Cx \wedge Ojx) \curlyvee \neg\exists x(Cx \wedge Ojx)) \wedge (\neg\exists x(Cx \wedge Ojx) \vee Px)$

(27) $\big(\exists x(Cx \wedge Ojx) \wedge (\neg\exists x(Cx \wedge Ojx) \vee Px)\big) \vee \big(\neg\exists x(Cx \wedge Ojx) \wedge (\neg\exists x(Cx \wedge Ojx) \vee Px)\big)$

And:[5]

$$\begin{aligned} [\![(26)]\!]^{M,g} &= \Big[\!\!\Big[\big(\exists x(Cx \wedge Ojx) \wedge (\neg\neg\exists x(Cx \wedge Ojx) \wedge Px)\big) \vee \bot\Big]\!\!\Big]^{M,g} \\ &= \Big\{h \mid g = h \text{ and } [\![\exists x(Cx \wedge Ojx) \wedge (\neg\neg\exists x(Cx \wedge Ojx) \wedge Px)]\!]^{M,h} \neq \emptyset\Big\} \\ &= \Big\{h \mid g = h \text{ and } [\![\exists x(Cx \wedge Ojx) \wedge Px]\!]^{M,h} \neq \emptyset\Big\} \end{aligned}$$

$$\begin{aligned} [\![(27)]\!]^{M,g} &= \Big[\!\!\Big[\big(\exists x(Cx \wedge Ojx) \wedge Px\big) \vee \neg\exists x(Cx \wedge Ojx)\Big]\!\!\Big]^{M,g} \\ &= \Big\{h \mid g = h \text{ and } [\![\exists x(Cx \wedge Ojx) \wedge Px]\!]^{M,h} \cup [\![\neg\exists x(Cx \wedge Ojx)]\!]^{M,h} \neq \emptyset\Big\} \\ &= \Big\{h \mid g = h \text{ and } \Big([\![\exists x(Cx \wedge Ojx) \wedge Px]\!]^{M,h} \neq \emptyset \text{ or } [\![\exists x(Cx \wedge Ojx)]\!]^{M,h} = \emptyset\Big)\Big\} \end{aligned}$$

As with (18), we can see that (27) is false just in case John owns a car but no car he owns is parked outside.

[5]Here, $[\![\bot]\!]^{M,g} := \emptyset$.

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