Copredication

quantificational issues and methodological implications

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Chomsky's question

Suppose the library has two copies of Tolstoy's War and Peace, *Peter takes out one, and John the other. Did Peter and John take out the same book, or different books? If we attend to the material factor of the lexical item, they took out different books; if we focus on its abstract component, they took out the same book. We can attend to both material and abstract factors simultaneously, as when we say that "the book that he is planning will weigh at least five pounds if he ever writes it," or "his book is in every store in the country."*

(Chomsky, 2000, p. 16)

Copredication

(1) The book that he is planning will weigh at least five pounds if he ever writes it.
information / abstract object + physical object

(2) Nobody understood the lecture, which lasted an hour.
information + event

(3) The bank was vandalized after calling in Bob's debt.
building + agent

(4) Lunch was delicious but took forever. (Asher, 2011, p. 11)
food + event

(5) London is so unhappy, ugly and polluted that it should be destroyed and rebuilt 100 miles away. (Chomsky, 2000, p. 37)
people + buildings + territory?

Issues

- **The philosophical issue**

  What, if anything, do the words ‘book’, ‘lecture’, ‘bank’, ‘lunch’ and ‘London’ refer to in sentences like (1)–(5) respectively?

- **The selectional issue**

  How can the selectional requirements of ‘understood’ and ‘lasted’ in (2), for example, be jointly satisfied by a single argument?

- **The quantificational issue**

  Some numerically quantified copredication sentences have truth conditions that are difficult to account for.

Outline

Quantification and individuation in copredication

(forthcoming in the *Journal of Semantics*)

Examples

(6) Peter read three books.

(7) Three books are heavy.

(8) Peter read three heavy books.

Situation 1

(suppose Peter read *FH*, *TKS* and *TC*, and v_1 is heavy)

volume 1	*Family Happiness* *The Kreutzer Sonata* *The Cossacks*

- Physically: 1 book. Informationally: 3 books.
- (6): True, (7),(8): False

(6) Peter read three books. ✓

(7) Three books are heavy. ×

(8) Peter read three heavy books. ×

Situation 2

(suppose Peter read *W&P*, and v_1, v_2 and v_3 are heavy)

volume 1 | *War and Peace*

volume 2 | *War and Peace*

volume 3 | *War and Peace*

- Physically: 3 books. Informationally: 1 books.
- (7): True, (6),(8): False

(6) Peter read three books. ×

(7) Three books are heavy. ✓

(8) Peter read three heavy books. ×

The third criterion

Situation 1			Situation 2	
volume 1	*Family Happiness* *The Kreutzer Sonata* *The Cossacks*		volume 1	*War and Peace*
			volume 2	*War and Peace*
			volume 3	*War and Peace*

(8) Peter read three heavy books. × ×

Key points

1. Nouns supporting copredication denote sets of complex objects—in the case of 'book', objects that have a part that is a physical volume and a part that is an informational (abstract) book.
2. Predicates encode criteria of individuation as part of their meaning.
3. Quantifiers access, compose and exploit criteria of individuation.

Complex objects

Suppose that we combine the books in situations 1 and 2 like this:

Situation 3

volume 1	*Family Happiness* *The Kreutzer Sonata* *The Cossacks*	volume 2	*War and Peace*
		volume 3	*War and Peace*
		volume 4	*War and Peace*

set of books in situation 3:

$\{\mathrm{v}_1{+}FH,\ \mathrm{v}_1{+}TKS,\ \mathrm{v}_1{+}TC,\ \mathrm{v}_2{+}W\&P,\ \mathrm{v}_3{+}W\&P,\ \mathrm{v}_4{+}W\&P\}$[1]

Problem: In this view, there are 6 books in situation 3.

Solution: This set of 6 is never used in plural quantification because of restrictions imposed by determiners.

[1] $a + b$ is a single object of which a and b are parts.

Target truth conditions

(6) Peter read three books.

'There is a plurality p of three books such that:

- Peter read every singular object in p, and
- no two distinct singular objects in p are informationally equivalent to each other.'

(7) Three books are heavy.

'There is a plurality p of three books such that:

- Every singular object in p is heavy, and
- no two distinct singular objects in p are physically equivalent to each other.'

(8) Peter read three heavy books.

'There is a plurality *p* of three books such that:

- Peter read every singular object in *p*,
- every singular object in *p* is heavy, and
- no two distinct singular objects in *p* are physically *or* informationally equivalent to each other.'

Criteria of individuation

Say that

- two objects are 'physically equivalent' if and only if their physical parts are identical, and
- a plurality is 'physically compressible' if and only if it includes two distinct objects that are physically equivalent to each other.

For example, (9) is physically compressible, because $v_1 + FH$ is physically equivalent to $v_1 + TKS$.[2]

(9) $v_1 + FH \oplus v_1 + TKS \oplus v_2 + W\&P$

(10) $v_1 + OMF \oplus v_2 + W\&P \oplus v_3 + W\&P$

(10) isn't physically compressible, but it is informationally compressible

[2] $a \oplus b$ is a plurality containing a and b. $+$ binds more tightly than $\oplus$.

Physical equivalence **phys-equiv** : $e \to (e \to t)$, abbreviated $\textsc{phys}$

Informational equivalence **info-equiv** : $e \to (e \to t)$, abbreviated $\textsc{info}$

Plurality x is compressible by relation R $\mathbf{comp}(x)(R)$

x is physically compressible $\mathbf{comp}(x)(\textsc{phys})$

x is (physically or informationally) compressible

$\mathbf{comp}(x)(\textsc{phys} \sqcup \textsc{info})$

$\sqcup$ is generalized disjunction (Partee and Rooth, 1983), e.g.

$$R_{e\to(e\to t)} \sqcup S_{e\to(e\to t)} \equiv \lambda x_e.\lambda y_e.R(x)(y) \vee S(x)(y)$$

and $\sqcap$ is generalized conjunction, e.g.

$$R_{e\to(e\to t)} \sqcap S_{e\to(e\to t)} \equiv \lambda x_e.\lambda y_e.R(x)(y) \wedge S(x)(y)$$

Formally:

$$\mathbf{comp}(x_e)(R_{e\to(e\to t)}) \stackrel{\text{df}}{=} \exists y_e.\exists z._e.y \neq z \wedge y \leq_i x \wedge z \leq_i x \wedge R(y)(z)$$

Therefore:

$$\mathbf{comp}(x)(\textsc{phys}) \equiv \exists y_e.\exists z_e.y \neq z \wedge y \leq_i x \wedge z \leq_i x \wedge \textbf{phys-equiv}(y)(z)$$

$$\mathbf{comp}(x)(\textsc{phys} \sqcup \textsc{info}) \equiv$$
$$\exists y_e.\exists z_e.y \neq z \wedge y \leq_i x \wedge z \leq_i x \wedge (\textbf{phys-equiv}(y)(z) \vee \textbf{info-equiv}(y)(z))$$

Novel 'lexical' entries

($\mathcal{R}$ abbreviates $e{\to}(e{\to}t)$)

(11) $\textit{book} \mapsto \lambda x_e\big(\mathbf{book}(x)\ ,\ \textsc{phys} \sqcap \textsc{info}\big)$

(12) $\textit{books} \mapsto \lambda x_e\big({}^*\mathbf{book}(x)\ ,\ \textsc{phys} \sqcap \textsc{info}\big)$

(13) $\textit{be heavy}_{pl} \mapsto \lambda y_e\big({}^*\mathbf{heavy}(y)\ ,\ \textsc{phys}\big)$

(14) $\textit{heavy}_{pl} \mapsto$

$$\lambda P_{e\to(t\times\mathcal{R})}.\lambda y_e\big((\pi_1(P(y)) \wedge {}^*\mathbf{heavy}(y))\ ,\ \pi_2(P(y)) \sqcup \textsc{phys}\big)$$

(15) $[\lambda_1\ \textit{Peter read}\ t_1] \mapsto \lambda v_e\big(\mathbf{read}(v)(\mathbf{p})\ ,\ \textsc{info}\big)$

$$\pi_1(a, b) = a \qquad \pi_2(a, b) = b$$

Quantification

(16) *three* $\mapsto$

$$\lambda P_{e\to(t\times\mathcal{R})}.\lambda Q_{e\to(t\times\mathcal{R})}\Big(\exists x_e\big(\,|x|\geq 3\wedge\pi_1(P(x))\wedge\pi_1(Q(x))\wedge\neg\mathbf{comp}(x)(\pi_2(P(x))\sqcup\pi_2(Q(x)))\big)\,,\ \pi_2(P(x))\sqcap\pi_2(Q(x))\Big)$$

(17) $\therefore$ *three books* $\mapsto$ (16)[(12)]

$$=\lambda Q_{e\to(t\times\mathcal{R})}\Big(\exists x_e\big(\,|x|\geq 3\wedge{}^{*}\mathbf{book}(x)\wedge\pi_1(Q(x))\wedge\neg\mathbf{comp}(x)((\textsc{phys}\sqcap\textsc{info})\sqcup\pi_2(Q(x)))\big)\,,\ (\textsc{phys}\sqcap\textsc{info})\sqcap\pi_2(Q(x))\Big)$$

Informational individuation

$$\textit{Peter read three books} \mapsto (17)[(15)]$$

$$= \Big(\exists x_e\big(\,|x| \geq 3 \wedge {}^{*}\mathbf{book}(x) \wedge \mathbf{read}(x)(\mathbf{p}) \wedge \neg\mathbf{comp}(x)((\textsc{phys} \sqcap \textsc{info}) \sqcup \textsc{info})\big)\,, (\textsc{phys} \sqcap \textsc{info}) \sqcap \textsc{info}\Big)$$

(18) $$= \Big(\exists x_e\big(\,|x| \geq 3 \wedge {}^{*}\mathbf{book}(x) \wedge \mathbf{read}(x) \wedge \neg\mathbf{comp}(x)(\textsc{info})\big)\,, \textsc{phys} \sqcap \textsc{info}\Big)$$

'There is a plurality *p* of three books such that:

- Peter read every singular object in *p*, and
- no two distinct singular objects in *p* are informationally equivalent to each other.'

Physical individuation

$$\textit{three books are heavy} \mapsto (17)[(13)]$$

$$= \Big(\exists x_e \big(|x| \geq 3 \wedge {}^*\textbf{book}(x) \wedge {}^*\textbf{heavy}(x) \wedge \neg\textbf{comp}(x)((\text{PHYS} \sqcap \text{INFO}) \sqcup \text{PHYS})\big) , (\text{PHYS} \sqcap \text{INFO}) \sqcap \text{PHYS}\Big)$$

(19) $$= \Big(\exists x_e \big(|x| \geq 3 \wedge {}^*\textbf{book}(x) \wedge {}^*\textbf{heavy}(x) \wedge \neg\textbf{comp}(x)(\text{PHYS})\big) , \text{PHYS} \sqcap \text{INFO}\Big)$$

'There is a plurality *p* of three books such that:

- every singular object in *p* is heavy, and
- no two distinct singular objects in *p* are physically equivalent to each other.'

Copredication

(20) *heavy books* $\mapsto$ (14)[(12)]

$$= \lambda y_e\big((^*\mathbf{book}(y) \wedge {}^*\mathbf{heavy}(y))\ ,\ (\text{PHYS} \sqcap \text{INFO}) \sqcup \text{PHYS}\big)$$

$$= \lambda y_e\big((^*\mathbf{book}(y) \wedge {}^*\mathbf{heavy}(y))\ ,\ \text{PHYS}\big)$$

(21) *three heavy books* $\mapsto$ (16)[(20)]

$$= \lambda Q_{e\to(t\times\mathcal{R})}\Big(\exists x_e\big(\,|x| \geq 3 \wedge {}^*\mathbf{book}(x) \wedge {}^*\mathbf{heavy}(x) \wedge \pi_1(Q(x)) \wedge \neg\mathbf{comp}(x)(\text{PHYS} \sqcup \pi_2(Q(x)))\big)\ ,\ \text{PHYS} \sqcap \pi_2(Q(x))\Big)$$

$$
\textit{Peter read three heavy books} \mapsto (21)[(15)]
$$

$$
= \Big(\exists x_e \big(\, |x| \geq 3 \wedge {}^{*}\mathbf{book}(x) \wedge {}^{*}\mathbf{heavy}(x) \wedge \mathbf{read}(x)(\mathbf{p}) \\
\wedge \neg \mathbf{comp}(x)(\textsc{phys} \sqcup \textsc{info}) \big) \ , \\
\textsc{phys} \sqcap \textsc{info} \Big)
$$

'There is a plurality p of three books such that:

- Peter read every singular object in p,
- every singular object in p is heavy, and
- no two distinct singular objects in p are physically *or* informationally equivalent to each other.'

Comparison with other approaches

This account makes three different principles of individuation available for (6)–(8):

1. Physical individuation (for (7)), requiring physical distinctness
2. Informational individuation (for (6)), requiring information distinctness
3. Copredicational individuation (for (8)), requiring both physical and informational distinctness

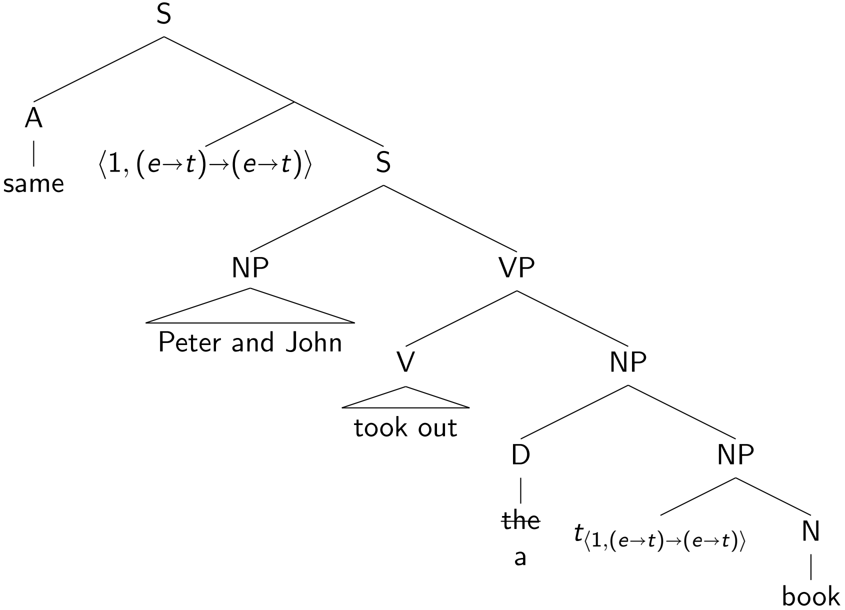
In contrast, Asher, (2011) and Cooper, (2011) only make 1 and 2 available, while Chatzikyriakidis and Luo, (2015) only makes 3 available.

The same, or different?

Did Peter and John take out the same book, or different books?

(Chomsky, 2000, p. 16)

- The semantics of 'same' (and 'different') is very tricky (Barker, 2007).
- That said, I imagine something like this:


S
A
same
⟨1, (e→t)→(e→t)⟩
S
NP
Peter and John
VP
V
took out
NP
D
~~the~~
a
NP
$t_{\langle 1,(e\to t)\to(e\to t)\rangle}$
N
book

Physical individuation:

$$same \mapsto \lambda V_{((e\to t)\to(e\to t))\to t}.\exists M_{(e\to t)\to(e\to t)}.\mathbf{mod}(\textsc{phys})(M) \wedge V(M)$$

Informational individuation:

$$same \mapsto \lambda V_{((e\to t)\to(e\to t))\to t}.\exists M_{(e\to t)\to(e\to t)}.\mathbf{mod}(\textsc{info})(M) \wedge V(M)$$

Where

$$\mathbf{mod}(R_{e\to(e\to t)})(M_{(e\to t)\to(e\to t)}) \stackrel{\text{df}}{=}$$
$$\forall P_{e\to t}.\forall x_e.M(P)(x) \to \Big(P(x) \wedge \forall y_e.M(P)(y) \to R\big(M(P)(x)\big)\big(M(P)(y)\big)\Big)$$

So for example, in words: $\mathbf{mod}(\textsc{phys})$ is true of a modifier M if and only if for any set P, $M(P) \subseteq P$ and all members of $M(P)$ are physically equivalent to each other.

(22) Peter and John took out the same book.

Physical individuation:

('we attend to the material factor of the lexical item')

$$\exists M_{(e\to t)\to(e\to t)}.\mathbf{mod}(\textsc{phys})(M) \wedge {}^{*}\big(\lambda x_e.\exists y_e.M(\mathbf{book})(y) \wedge \mathbf{take}(y)(x)\big)(\mathbf{p}\oplus\mathbf{j})$$

Informational individuation: ('we focus on its abstract component')

$$\exists M_{(e\to t)\to(e\to t)}.\mathbf{mod}(\textsc{info})(M) \wedge {}^{*}\big(\lambda x_e.\exists y_e.M(\mathbf{book})(y) \wedge \mathbf{take}(y)(x)\big)(\mathbf{p}\oplus\mathbf{j})$$

Philosophical/methodological implications

Internalism and externalism about semantics

Collins, (2011):

> Linguistic externalism*: The explanations offered by successful linguistic theory (broadly conceived) entail or presuppose externalia (objects or properties individuated independent of speaker-hearers' cognitive states). The externalia include the quotidian objects we take ourselves to talk about each day.*
>
> Linguistic internalism*: The explanations offered by successful linguistic theory neither presuppose nor entail externalia. There are externalia, but they do not enter into the explanations of linguistics qua externalia. Linguistics is methodologically solipsistic; its kinds are internalist.*

Collins against externalism

because of copredication

> *Bill took a decade to write the book and was happy when it arrived from the publishers weighing 2lb, and even happier when it sold out in the first week.*

We do not imagine that there is one thing that Bill took a decade to write, weighs 2lb, and was sold out in the first week. [...] ontological quandaries appear to have nothing whatsoever to do with our semantic competence [...] We *use words to talk about things in a range of complex ways, whose coherence or not appears to be independent of the status of the objects talked about.*

(Collins, 2009, pp. 58–59)

Chomsky against externalism

because of copredication

> *Contemporary philosophy of language [...] asks to what a word refers, giving various answers. But the question has no clear meaning. The example of "book" is typical. It makes little sense to ask to what* thing *the expression "Tolstoy's* War and Peace*" refers, when Peter and John take identical copies out of the library. The answer depends on how the semantic features are used when we think and talk, one way or another. In general, a word, even of the simplest kind, does not pick out an entity of the world, or of our "belief space".*

(Chomsky, 2000, p. 17)

The form of the argument

1. If ‘book’ refers to anything, those things must be both abstract/informational and concrete/physical.
2. Nothing is both abstract/informational and concrete/physical.
3. Therefore, ‘book’ does not refer to anything.

(And *mutatis mutandis* for other nouns supporting copredication, e.g. ‘lunch’, ‘bank’, ‘lecture’, ‘London’. . .)

It seems to me that the strength of this argument depends on whether or not you're willing to countenance physical+informational composite objects among the 'entit[ies] of the world, or of our "belief space"'.

> *I doubt that people think that among the constituents of the world are entities that are simultaneously abstract and concrete (like books and banks)*

(Chomsky, 2003, p. 290)

Are we just left trading intuitions?

Possible further assumptions

- Other than causing philosophical problems for externalists, there is nothing semantically special about nouns supporting copredication.
- Therefore, it would be ill-motivated to make special allowances for them in order to salvage externalism.

But. . .

- The physical+informational objects are motivated not (primarily) by the need to solve ontological quandaries, but in order to get the facts right about the truth conditions of numerically-quantified copredication sentences.
- Do internalists think that there *are* truth conditions, or that speaker truth-value judgements are something that semantic theory should predict? There appears to be some disagreement (Collins, 2009 vs. Pietroski, 2005).
- Either way, there appears to be agreement that *entailments* are something that a semantic theory should predict.

(23) John picked up three books.
John memorized every book.
- - - - - - - - - - - - - - - - - - - -
$\not\therefore$ John memorized three books.

$$\exists x_e\big(\,|x| \geq 3 \wedge {}^*\mathbf{book}(x) \wedge \mathbf{pick\text{-}up}(x)(\mathbf{j}) \wedge \neg\mathbf{comp}(x)(\textsc{phys})\big)$$
$$\forall x_e\big(\mathbf{book}(x) \rightarrow \mathbf{memorize}(x)(\mathbf{j})\big)$$
- -
$$\nvdash \exists x_e\big(\,|x| \geq 3 \wedge {}^*\mathbf{book}(x) \wedge \mathbf{memorize}(x)(\mathbf{j}) \wedge \neg\mathbf{comp}(x)(\textsc{info})\big)$$

(24) John picked up three books.
John defaced every book.

$\therefore$ John defaced three books.

$$\exists x_e\big(\,|x| \geq 3 \wedge {}^*\mathbf{book}(x) \wedge \mathbf{pick\text{-}up}(x)(\mathbf{j}) \wedge \neg\mathbf{comp}(x)(\textsc{phys})\big)$$
$$\forall x_e\big(\mathbf{book}(x) \rightarrow \mathbf{deface}(x)(\mathbf{j})\big)$$

$$\vdash \exists x_e\big(\,|x| \geq 3 \wedge {}^*\mathbf{book}(x) \wedge \mathbf{deface}(x)(\mathbf{j}) \wedge \neg\mathbf{comp}(x)(\textsc{phys})\big)$$

- You wouldn't get the non-entailment of (23), and the contrast with (24), by assuming that 'book' is just like other nouns.
- In other words, (23)–(24) show that there is something semantically unusual about 'book'.
- What other clue is there that 'book' is semantically unusual? The copredication puzzles!

Assuming the referentialist doctrine [raises the problem of copredication]. It seems then that we must abandon it in this case. If we do, the problem dissolves.

(Chomsky, 2013, p. 41)

- My contention: that is not a good thing!
- Taking the problems raised by copredication seriously as problems can lead to important insights.

Methodological externalism

- We should make an effort to keep semantic theory externalistically viable (given a suitably generous conception of what is externalistically viable) *even if* thoroughgoing externalism is unsustainable in the long run.
- Not just for copredication:

(25) The average American has 2.3 children.

Kennedy and Stanley, (2009):

$$\frac{\sum_{\mathbf{american}(x)} max\,\{d : \exists v((^{*}\mathbf{child}(v) \wedge |v| = d) \wedge \mathbf{have}(v)(x))\}}{|\{y : \mathbf{american}(y)\}|} = 2.3$$

(26) Americans have 2.3 children on average.

Conclusion

- One of the challenges that copredication poses to linguistic theory concerns quantification: different predicates can impose different criteria of individuation on their arguments.
- This challenge can be met by:
  - Defining criteria of individuation as equivalence relations on subsets of the domain of discourse.
  - Incorporating them into lexical entries.
  - Allowing determiners to exploit them.
- Copredication is also a factor in a debate about what the philosophical commitments of our semantic theories are (or should be).

Semanticists should proceed *as if* Kennedy and Stanley, (2009, p. 584) are right:

> *semantic theory [. . .] can tell us what the costs would be of denying the existence of certain kinds of entities [. . .] If a straightforward semantic theory for arithmetic is true, then a sentence such as 'There is a prime number between two and five' entails the existence of numbers. As a result, a nominalist who rejects the existence of numbers is committed either to rejecting the simple semantics, or to rejecting the truth of 'There is a prime number between two and five.'*

. . . and similarly for 'book' and 'bank', etc.

Thanks!

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In the full system things are a bit more complex, e.g. we really have

$$books \mapsto \lambda y_e\,(^*\mathbf{book}(y)\;,\;\lambda f_{e\to\mathcal{R}}.f(y) \sqsubseteq (\text{PHYS} \sqcap \text{INFO}))$$

where $\sqsubseteq$ is generalized entailment, such that for example

$$R_{e\to(e\to t)} \sqsubseteq S_{e\to(e\to t)} \equiv \forall x_e.\forall y_e.R(x)(y) \to S(x)(y)$$

Lexical entries for higher-arity predicates can be set up accordingly, e.g.

$$\begin{aligned} read \mapsto \lambda y_e.\lambda z_e\big(&\mathbf{read}(y)(z)\;, \\ &\lambda g_{e\to\mathcal{R}}.g(y) \sqsubseteq \text{INFO} \wedge g(z) \sqsubseteq \text{ANI}\big) \end{aligned}$$

three $\mapsto$

$$\lambda P_{e\to T}.\lambda Q_{e\to T}\Big(\ \exists x_e\big(\,|x| \geq 3 \wedge \pi_1(P(x)) \wedge \pi_1(Q(x)) \wedge \neg\mathbf{comp}(x)(\Omega(\lambda v_e.\pi_2(P(v))) \sqcup \Omega(\lambda v_e.\pi_2(Q(v))))\big)\,, \lambda h_{e\to\mathcal{R}}.\exists v_e.\pi_1(P(v)) \wedge \pi_2(P(v))(h) \wedge \pi_2(Q(v))(h)\Big)$$

Where T abbreviates $(e\to\mathcal{R})\to t$ and the Ω function is defined as follows:

$$\Omega_{(e\to T)\to\mathcal{R}}(A_{e\to T}) \stackrel{\text{df}}{=} \bigsqcup\{R : \exists x_e \exists f_{e\to\mathcal{R}}(A(x)(f) \wedge f(x) = R)\}$$

This is just a way of accessing the pseudo-equivalence relation associated with the abstracted variable, e.g. of accessing $\text{PHYS} \sqcap \text{INFO}$ given the lexical entry for 'book'.

Peter read three books $\mapsto$

$$\Big(\exists x_e\big(\,|(|\,x) \geq 3 \wedge {}^*\mathbf{book}(x) \wedge \mathbf{read}(x)(\mathbf{p})$$
$$\wedge\neg\mathbf{comp}(x)\big(\Omega(\lambda v_e.\lambda f_{e\to\mathcal{R}}.f(v) \sqsubseteq (\text{PHYS} \sqcap \text{INFO}))$$
$$\sqcup\, \Omega(\lambda v_e.\lambda f_{e\to\mathcal{R}}.f(v) \sqsubseteq \text{INFO} \wedge f(\mathbf{p}) \sqsubseteq \text{ANI})\big)\big)\ ,$$
$$\lambda h_{e\to\mathcal{R}}.\exists v_e.{}^*\mathbf{book}(v) \wedge h(v) \sqsubseteq (\text{PHYS} \sqcap \text{INFO})$$
$$\wedge\, h(v) \sqsubseteq \text{INFO} \wedge h(\mathbf{p}) \sqsubseteq \text{ANI}\Big)$$

$$\equiv \Big(\exists x_e\big(\,|(|\,x) \geq 3 \wedge {}^*\mathbf{book}(x) \wedge \mathbf{read}(x)(\mathbf{p})$$
$$\wedge\, \neg\mathbf{comp}(x)((\text{PHYS} \sqcap \text{INFO}) \sqcup \text{INFO})\big)\ ,$$
$$\lambda h_{e\to\mathcal{R}}.\exists v_e.{}^*\mathbf{book}(v) \wedge h(v) \sqsubseteq (\text{PHYS} \sqcap \text{INFO}) \wedge h(\mathbf{p}) \sqsubseteq \text{ANI}\Big)$$

$$\equiv \Big(\exists x_e\big(\,|(|\,x) \geq 3 \wedge {}^*\mathbf{book}(x) \wedge \mathbf{read}(x)(\mathbf{p}) \wedge \neg\mathbf{comp}(x)(\text{INFO})\big)\ ,$$
$$\lambda h_{e\to\mathcal{R}}.\exists v_e.{}^*\mathbf{book}(v) \wedge h(v) \sqsubseteq (\text{PHYS} \sqcap \text{INFO}) \wedge h(\mathbf{p}) \sqsubseteq \text{ANI}\Big)$$

heavy $\mapsto$

$$\lambda P_{e\to T}.\lambda x_e\Big((\pi_1(P(x)) \wedge {}^*\textbf{heavy}(x))\ ,\\ \lambda f_{e\to\mathcal{R}}.\exists g_{e\to\mathcal{R}}\big(\pi_2(P(x))(g) \wedge f \sim_x g\\ \wedge f(x) \sqsubseteq \big(\text{PHYS} \sqcup \Omega(\lambda v_e.\pi_2(P(v)))\big)\big)\Big)$$

$\therefore$ *heavy books* $\mapsto$

$$\lambda x_e\Big(({}^*\textbf{book}(x) \wedge {}^*\textbf{heavy}(x))\ ,\\ \lambda f_{e\to\mathcal{R}}.\exists g_{e\to\mathcal{R}}\big(g(x) \sqsubseteq (\text{PHYS} \sqcap \text{INFO}) \wedge f \sim_x g\\ \wedge f(x) \sqsubseteq \big(\text{PHYS} \sqcup (\text{PHYS} \sqcap \text{INFO})\big)\big)\Big)$$

$$= \lambda x_e\big(({}^*\textbf{book}(x) \wedge {}^*\textbf{heavy}(x))\ ,\ \lambda f_{e\to\mathcal{R}}.f(x) \sqsubseteq \text{PHYS}\big)$$