Quantificational subordination as anaphora to a function

Matthew Gotham
University of Oxford

24th Conference on Formal Grammar, University of Latvia
11 August 2019

Outline

Background

Outline

Outline

- Background
- Outline of the proposal
- Examples
  - Refset anaphora
  - Telescoping

Outline

Background

Quantificational subordination (QS)

(1) If you give every child a present, some child will open it. (Ranta 1994)

(2) Every student bought a book. Most of them read it.

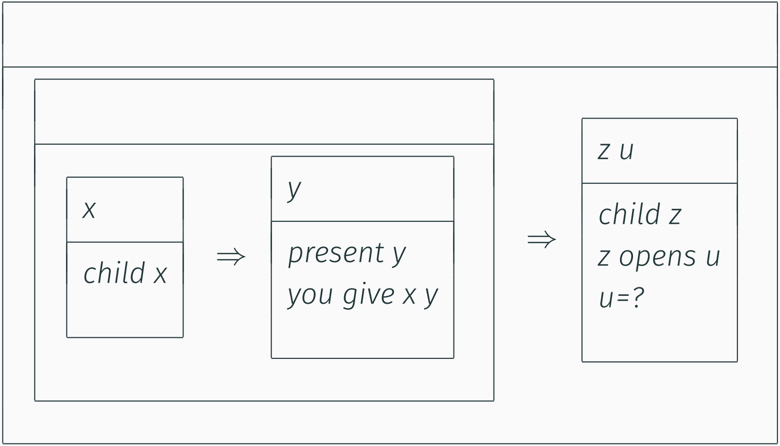
(3) Every player chooses a pawn. He puts it on square one. (Groenendijk & Stokhof 1991)

Examples like (3) are often called ‘telescoping’.

Pronouns inaccessible in first-generation dynamic semantics

E.g. DRT:

If you give every child a present, some child will open it.



Pronouns inaccessible in first-generation dynamic semantics

Every player chooses a pawn. He puts it on square one.

z u
[*x* | *player x*] ⇒ [*y* | *pawn y*, *x chooses y*]
z puts u on square one
z=?
u=?

Second-generation dynamic semantics

Via a generalization to using sets of assignments:

$$\llbracket \textit{every}^x \textit{ player chooses } a^y \textit{ pawn} \rrbracket =$$

$$\{ \langle F, H\rangle \mid \exists G : (\forall f \in F : \exists g \in G : f \approx_x g \;\&\; \forall g \in G : \exists f \in F : f \approx_x g)$$

$$\& \; \{g(x) \mid g \in G\} = \llbracket \textit{player} \rrbracket$$

$$\& \; (\forall g \in G : \exists h : g \approx_y h \;\&\; h(y) \in \llbracket \textit{pawn} \rrbracket \;\&\; \langle h(x), h(y)\rangle \in \llbracket \textit{choose} \rrbracket)$$

$$\& \; H = \{h \mid \exists g \in G : g \approx_y h \;\&\; h(y) \in \llbracket \textit{pawn} \rrbracket \;\&\; \langle h(x), h(y)\rangle \in \llbracket \textit{choose} \rrbracket\} \}$$

Second-generation dynamic semantics

Via a generalization to using sets of assignments:

$$[\![\textit{every}^x \textit{ player chooses } a^y \textit{ pawn}]\!] =$$

$$\{ \langle F, H\rangle \mid \exists G : (\forall f \in F : \exists g \in G : f \approx_x g \;\&\; \forall g \in G : \exists f \in F : f \approx_x g)$$
$$\& \; \{g(x) \mid g \in G\} = [\![\textit{player}]\!]$$
$$\& \; (\forall g \in G : \exists h : g \approx_y h \;\&\; h(y) \in [\![\textit{pawn}]\!] \;\&\; \langle h(x), h(y)\rangle \in [\![\textit{choose}]\!])$$
$$\& \; H = \{h \mid \exists g \in G : g \approx_y h \;\&\; h(y) \in [\![\textit{pawn}]\!] \;\&\; \langle h(x), h(y)\rangle \in [\![\textit{choose}]\!]\} \}$$

- I.e., $\{h(x) \mid h \in H\}$ is the set of players, and for every $h \in H, h(y)$ is a pawn chosen by $h(x)$. H therefore encodes the necessary dependency between pawns and players.

Second-generation dynamic semantics

Via a generalization to using sets of assignments:

$$[\![\textit{every}^x \textit{ player chooses } a^y \textit{ pawn}]\!] =$$

$$\{\, \langle F, H\rangle \mid \exists G : (\forall f \in F : \exists g \in G : f \approx_x g \;\&\; \forall g \in G : \exists f \in F : f \approx_x g)$$
$$\&\; \{g(x) \mid g \in G\} = [\![\textit{player}]\!]$$
$$\&\; (\forall g \in G : \exists h : g \approx_y h \;\&\; h(y) \in [\![\textit{pawn}]\!] \;\&\; \langle h(x), h(y)\rangle \in [\![\textit{choose}]\!])$$
$$\&\; H = \{h \mid \exists g \in G : g \approx_y h \;\&\; h(y) \in [\![\textit{pawn}]\!] \;\&\; \langle h(x), h(y)\rangle \in [\![\textit{choose}]\!]\} \,\}$$

- I.e., $\{h(x) \mid h \in H\}$ is the set of players, and for every $h \in H, h(y)$ is a pawn chosen by $h(x)$. H therefore encodes the necessary dependency between pawns and players.
- It's quite a complex and roudabout way to get to that dependency, though.

Type-theoretical semantics (TTS) suggests an answer

Type-theoretical semantics (TTS) suggests an answer

- Propositions-as-types principle

Type-theoretical semantics (TTS) suggests an answer

- Propositions-as-types principle
- $(\Sigma x : A)B$—the type of ordered pairs $\langle a, b\rangle$, where $a : A$ and $b : B[a/x]$

Type-theoretical semantics (TTS) suggests an answer

- Propositions-as-types principle
- $(\Sigma x : A)B$—the type of ordered pairs $\langle a, b \rangle$, where $a : A$ and $b : B[a/x]$
- $(\Pi x : A)B$—the type of functions with domain A such that, for any $a : A$, $f(a) : B[a/x]$

Type-theoretical semantics (TTS) suggests an answer

- Propositions-as-types principle
- $(\Sigma x : A)B$—the type of ordered pairs $\langle a, b\rangle$, where $a : A$ and $b : B[a/x]$
- $(\Pi x : A)B$—the type of functions with domain A such that, for any $a : A$, $f(a) : B[a/x]$

(1) If you give every child a present, some child will open it.

$$\big(\Pi f : \big(\Pi u : (\Sigma x : e)\,\textsc{child}(x)\big)\\ (\Sigma v : (\Sigma y : e)\,\textsc{present}(y))\,\textsc{give}(\text{you}', \pi_1(v), \pi_1(u))\big)\\ (\Sigma w : (\Sigma z : e)\,\textsc{child}(z))\,\textsc{open}(\pi_1(w), \pi_1(\pi_1(f(w))))$$

Using the function

$$\big(\Pi f : \big(\Pi u : (\Sigma x : e)\ \textsc{child}(x)\big)$$
$$(\Sigma v : (\Sigma y : e)\ \textsc{present}(y))\ \textsc{give}(\textsc{you}', \pi_1(v), \pi_1(u))\big)$$
$$(\Sigma w : (\Sigma z : e)\ \textsc{child}(z))\ \textsc{open}(\pi_1(w), \pi_1(\pi_1(f(w))))$$

Using the function

$$\big(\Pi f : \big(\Pi u : (\Sigma x : e)\,\textsc{child}(x)\big)$$
$$(\Sigma v : (\Sigma y : e)\,\textsc{present}(y))\,\textsc{give}(\text{you}', \pi_1(v), \pi_1(u))\big)$$
$$(\Sigma w : (\Sigma z : e)\,\textsc{child}(z))\,\textsc{open}(\pi_1(w), \pi_1(\pi_1(f(w))))$$

- *you give every child a present* $\rightsquigarrow$ a function f mapping every child to a present you give him/her.
- *some child will open it* $\rightsquigarrow$ a child z and a proof that you open $f(z)$.

Using the function

$$\big(\Pi f : \big(\Pi u : (\Sigma x : e)\,\textsc{child}(x)\big)\\ (\Sigma v : (\Sigma y : e)\,\textsc{present}(y))\,\textsc{give}(\text{you}', \pi_1(v), \pi_1(u))\big)\\ (\Sigma w : (\Sigma z : e)\,\textsc{child}(z))\,\textsc{open}(\pi_1(w), \pi_1(\pi_1(f(w))))$$

- *you give every child a present* ⇝ a function f mapping every child to a present you give him/her.
- *some child will open it* ⇝ a child z and a proof that you open $f(z)$.
- The fact that the first sentence expresses a function makes this kind of dependency possible.

Using the function

$$\big(\Pi f : \big(\Pi u : (\Sigma x : e)\, \textsc{child}(x)\big)\; (\Sigma v : (\Sigma y : e)\, \textsc{present}(y))\, \textsc{give}(\text{you}', \pi_1(v), \pi_1(u))\big)\; (\Sigma w : (\Sigma z : e)\, \textsc{child}(z))\, \textsc{open}(\pi_1(w), \pi_1(\pi_1(f(w))))$$

- *you give every child a present* $\rightsquigarrow$ a function f mapping every child to a present you give him/her.
- *some child will open it* $\rightsquigarrow$ a child z and a proof that you open $f(z)$.
- The fact that the first sentence expresses a function makes this kind of dependency possible.
- BUT it is actually crucial that an appropriate argument to the function is overtly present in the second sentence.

Limitations

(3) Every player chooses a pawn. He puts it on square one.

Ranta (1994: 73):

> *the only way to interpret the text [...] is by treating the pronoun 'he' as an abbreviation of 'every player'*

Obviously, this 'abbreviation' strategy is unsatisfactory.

Limitations

(3) Every player chooses a pawn. He puts it on square one.

Ranta (1994: 73):

> *the only way to interpret the text […] is by treating the pronoun 'he' as an abbreviation of 'every player'*

Obviously, this 'abbreviation' strategy is unsatisfactory.

(2) Every student bought a book. Most of them read it.

No mechanism for plural anaphora (yet).

Outline of the proposal

Witness semantics (Gotham 2018)

Idea: take the ideas of TTS (dependent pairs/functions) and apply them in (sort of) simple type theory.

Idea: take the ideas of TTS (dependent pairs/functions) and apply them in (sort of) simple type theory.

(1) If you give every child a present, some child will open it.

$$\exists f.\forall g.\big(\forall x.\mathrm{child}'x \rightarrow \big(\mathrm{present}'(gx)_0 \wedge \mathrm{give}'(\mathrm{you}', (gx)_0, x, (gx)_1)\big)\big)$$
$$\rightarrow \big(\mathrm{child}'(fg)_0 \wedge \mathrm{open}'((fg)_0, (g(fg)_0)_0, (fg)_1)\big)$$

$$f : (e \rightarrow e \times v) \rightarrow e \times v \qquad g : e \rightarrow e \times v \qquad x : e$$

- We'll use events (type v) as the model-theoretic analogs of proofs objecs in TTS.
- (Notation: we have $._{0/1}$ for left/right projections, i.e. $(a, b)_0 = a$ and $(a, b)_1 = b$.)

Basic ideas

- Sentences denote relations, not between verifying assignments, but actual verifying <u>things</u>: entities, events and structures built up from them.

Basic ideas

- Sentences denote relations, not between verifying assignments, but actual verifying things: entities, events and structures built up from them.
- This requires the use of (parametric) polymorphism in type annotations, given by greek letters in what follows.

Basic ideas

- Sentences denote relations, not between verifying assignments, but actual verifying things: entities, events and structures built up from them.
- This requires the use of (parametric) polymorphism in type annotations, given by greek letters in what follows.
- Pronouns denote functions from input contexts to entities/sets.

Basic ideas

- Sentences denote relations, not between verifying assignments, but actual verifying things: entities, events and structures built up from them.
- This requires the use of (parametric) polymorphism in type annotations, given by greek letters in what follows.
- Pronouns denote functions from input contexts to entities/sets.
- Existential closure at the text level.

Extension to cover QS

- The version of Gotham 2018 doesn't do any better than TTS for QS.

Extension to cover QS

- The version of Gotham 2018 doesn't do any better than TTS for QS.
- This paper:
    - Revised lexical entries for quantificational determiners: a sentence headed by one denotes a function.
    - The domain of that function is the refset.
    - Both the function itself and its domain are targets for anaphora.
    - A mechanism for accessing the range of the function to account for telescoping.

Extension to cover QS

- The version of Gotham 2018 doesn't do any better than TTS for QS.
- This paper:
    - Revised lexical entries for quantificational determiners: a sentence headed by one denotes a function.
    - The domain of that function is the refset.
    - Both the function itself and its domain are targets for anaphora.
    - A mechanism for accessing the range of the function to account for telescoping.
- Also: an accompanying syntactic theory.

Syntactic theory

Categories:

$$A, B \quad ::= \quad \mathsf{S} \mid \mathsf{S}_{\sigma,\tau} \mid \mathsf{N}_{\sigma,\tau} \mid \mathsf{NP} \mid \mathsf{NP}_\sigma \mid \mathsf{NPL} \mid \mathsf{NPL}_\sigma \mid A/B \mid A\backslash B \mid A^B$$

where

$$\sigma, \tau \quad ::= \quad 1 \mid e \mid t \mid \sigma \to \tau \mid \sigma \times \tau$$

Type map:

- $\mathrm{Ty}(\mathsf{S}_{\sigma,\tau}) = \mathrm{Ty}(\mathsf{N}_{\sigma,\tau}) = \sigma \to \tau \to t$
- $\mathrm{Ty}(\mathsf{S}) = t$
- $\mathrm{Ty}(\mathsf{NP}_\sigma) = \sigma \to e$
- $\mathrm{Ty}(\mathsf{NP}) = e$
- $\mathrm{Ty}(\mathsf{NPL}_\sigma) = \sigma \to e \to t$
- $\mathrm{Ty}(\mathsf{NPL}) = e \to t$
- $\mathrm{Ty}(A\backslash B) = \mathrm{Ty}(A/B) = \mathrm{Ty}(A^B) = \mathrm{Ty}(B) \to \mathrm{Ty}(A)$

Syntactic theory

Combinatory rules:

$$\frac{f : B/A \quad a : A}{fa : B} >$$

$$\frac{f : A/B}{\lambda g.\lambda c.f(gc) : A^C/B^C}\ G$$

$$\frac{f : A\backslash B}{\lambda g.\lambda c.f(gc) : A^C\backslash B^C}\ G$$

$$\frac{a : A \quad f : B\backslash A}{fa : B} <$$

$$\frac{f : (A/B)^C}{\lambda b.\lambda c.fcb : A^C/B}\ X$$

$$\frac{f : (A\backslash B)^C}{\lambda b.\lambda c.fcb : A^C\backslash B}\ X$$

Partial type theory

For any types σ, τ and term $T : \sigma \to \tau$,

$\mathrm{dom} T := \lambda s^{\sigma}.Ts \neq \star^{\tau}$

where

$\star^{\beta}$ is stipulated for any base type β

and

$\star^{\sigma \times \tau} := (\star^{\sigma}, \star^{\tau})$
$\star^{\sigma \to \tau} :=$ the unique $f :: \sigma \to \tau$ such that for any $s :: \sigma, fs = \star^{\tau}$

Mini lexicon

input (left context), output (witness)

$$a \rightsquigarrow \lambda P^{\alpha\to e\to t}.\lambda V^{e\to\alpha\times e\to\beta\to t}.\lambda i^{\alpha}.\lambda u^{e\times\beta}.Piu_0 \wedge Vu_0(i,u_0)u_1$$

$$: (\mathsf{S}_{\alpha,e\times\beta}/(\mathsf{S}_{\alpha\times e,\beta}\backslash\mathsf{NP}))/\mathsf{N}_{\alpha,e}$$

$$det \rightsquigarrow \lambda P^{\alpha\to e\to t}.\lambda V^{e\to\alpha\times e\to\beta\to t}.\lambda i^{\alpha}.\lambda f^{e\to\beta}.\mathrm{dom} f \subseteq (Pi) \wedge \mathrm{det}'(Pi)(\mathrm{dom} f) \wedge \forall x^e.\mathrm{dom} f x \to Vx(i,x)(fx)$$

$$: (\mathsf{S}_{\alpha,e\to\beta}/(\mathsf{S}_{\alpha\times e,\beta}\backslash\mathsf{NP}))/\mathsf{N}_{\alpha,e}$$

$$book \rightsquigarrow \lambda i^{\alpha}.\mathrm{book}' : \mathsf{N}_{\alpha,e}$$

$$bought \rightsquigarrow \lambda D^{(e\to\alpha\to v\to t)\to\beta\to\gamma\to t}.\lambda x^e.D(\lambda y^e.\lambda i^{\alpha}.\lambda e^v.\mathrm{buy}'(x,y,e))$$

$$: (\mathsf{S}_{\beta,\gamma}\backslash\mathsf{NP})/(\mathsf{S}_{\beta,\gamma}/(\mathsf{S}_{\alpha,v}\backslash\mathsf{NP}))$$

$he,it \rightsquigarrow \lambda g^{\alpha\to e}.\lambda V^{e\to\alpha\to\beta\to t}.\lambda i^{\alpha}.V(gi)i : (\mathsf{S}_{\alpha,\beta}/(\mathsf{S}_{\alpha,\beta}\backslash\mathsf{NP}))^{\mathsf{NP}_\alpha}$

$of\ them \rightsquigarrow \lambda G^{\alpha\to e\to t}.\lambda i^{\alpha}.Gi : (\mathsf{N}_{\alpha,e})^{\mathsf{NPL}_\alpha}$

$; \rightsquigarrow \lambda p^{\alpha\to\beta\to t}.\lambda q^{\alpha\times\beta\to\gamma\to t}.\lambda i^{\alpha}.\lambda o^{\beta\times\gamma}.pio_0 \wedge q(i,o_0)o_1$

$\quad : (\mathsf{S}_{\alpha,\beta\times\gamma}/\mathsf{S}_{\alpha\times\beta,\gamma})\backslash\mathsf{S}_{\alpha,\beta}$

$[\text{close}] := \lambda p^{1\to\alpha\to t}.\exists a^{\alpha}.p{*}a : \mathsf{S}/\mathsf{S}_{1,\alpha}$

where $* : 1$

Examples

(2) Every student bought a book; most of them read it.

(2) Every student bought a book; most of them read it.

Resolved lexical entries:

(2) Every student bought a book; most of them read it.

Resolved lexical entries:

$$\textit{every} \rightsquigarrow \lambda P^{1\to e\to t}.\lambda V^{e\to 1\times e\to e\times v\to t}.\lambda i^{1}.\lambda f^{e\to e\times v}.\mathrm{dom} f = (Pi) \land \forall x^{e}.\mathrm{dom} fx \to Vx(i,x)(fx)$$

$$: (\mathrm{S}_{1,e\to e\times v}/(\mathrm{S}_{1\times e,e\times v}\backslash \mathrm{NP}))/\mathrm{N}_{1,e}$$

$\textit{student} \rightsquigarrow \lambda i^{1}.\mathrm{student}' : \mathrm{N}_{1,e}$ $\quad$ $\textit{book} \rightsquigarrow \lambda i^{1\times e}.\mathrm{book}' : \mathrm{N}_{1\times e,e}$

$\textit{bought} \rightsquigarrow$

$$\lambda D^{(e\to(1\times e)\times e\to v\to t)\to 1\times e\to e\times v\to t}.\lambda x^{e}.D(\lambda y^{e}.\lambda i^{(1\times e)\times e}.\lambda e^{v}.\mathrm{buy}'(x,y,e))$$

$$: (\mathrm{S}_{1\times e,e\times v}\backslash \mathrm{NP})/(\mathrm{S}_{1\times e,e\times v}/(\mathrm{S}_{(1\times e)\times e,v}\backslash \mathrm{NP}))$$

$$a \rightsquigarrow \lambda P^{1\times e\to e\to t}.\lambda V^{e\to(1\times e)\times e\to v\to t}.\lambda i^{1\times e}.\lambda u^{e\times v}.Piu_0 \land V(u_0)_0(i,u_0)u_1$$

$$: (\mathrm{S}_{1\times e,e\times v}/(\mathrm{S}_{(1\times e)\times e,v}\backslash \mathrm{NP}))/\mathrm{N}_{1\times e,e}$$

$; \rightsquigarrow$

$$\lambda p^{1\to(e\to e\times v)\to t}.\lambda q^{1\times(e\to e\times v)\to(e\to v)\to t}.\lambda i^{1}.\lambda o^{(e\to e\times v)\times(e\to v)}.pio_0 \land q(i,o_0)o_1$$

$$: (\mathrm{S}_{1,(e\to e\times v)\times(e\to v)}/\mathrm{S}_{1\times(e\to e\times v),e\to v})\backslash \mathrm{S}_{1,(e\to e\times v)}$$

First sentence derivation

$$\dfrac{\begin{array}{c}\textit{bought}\\ (S_{\dots}\backslash NP)/(S_{\dots}/(S_{\dots}\backslash NP))\end{array} \qquad \dfrac{\begin{array}{c}\textit{a}\\ (S_{\dots}/(S_{\dots}\backslash NP))/N_{\dots}\end{array} \quad \begin{array}{c}\textit{book}\\ N_{\dots}\end{array}}{(S_{\dots}/(S_{\dots}\backslash NP))}>}{S_{1\times e, e\times v}\backslash NP}>$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\begin{array}{c}\textit{every}\\ (S_{\dots}/(S_{\dots}\backslash NP))/N_{\dots}\end{array} \quad \begin{array}{c}\textit{student}\\ N_{\dots}\end{array}}{S_{\dots}/(S_{\dots}\backslash NP)}> \qquad \begin{array}{c}\textit{bought a book}\\ S_{\dots}\backslash NP\end{array}}{S_{{\color{blue}1},{\color{red}e\to e\times v}}}> \qquad \begin{array}{c};\\ (S_{\dots}/S_{\dots})\backslash S_{\dots}\end{array}}{\dfrac{S_{\dots}/S_{\dots}}{(S_{\dots})^{NPL_{\dots}}/(S_{\dots})^{NPL_{\dots}}}G}<}{((S_{1,(e\to e\times v)\times(e\to v)})^{NPL_{1\times(e\to e\times v)}})^{NP_{(1\times(e\to e\times v))\times e}}/((S_{\dots})^{NPL_{\dots}})^{NP_{\dots}}}G$$

(2) Every student bought a book; most of them read it.

(2) Every student bought a book; most of them read it.

Resolved lexical entries:

(2) Every student bought a book; most of them read it.

Resolved lexical entries:

most $\rightsquigarrow$

$$\lambda P^{1\times(e\to e\times v)\to e\to t}.\lambda V^{e\to(1\times(e\to e\times v))\to v\to t}.\lambda i^{1\times(e\to e\times v)}.\lambda f^{e\to v}.\mathrm{dom} f \subseteq (Pi) \\ \wedge \mathrm{most}'(Pi)(\mathrm{dom} f) \\ \wedge \forall x^{e}.\mathrm{dom} f x \to Vx(i,x)(fx)$$

$$: (\mathsf{S}_{1\times(e\to e\times v),e\to v}/(\mathsf{S}_{(1\times(e\to e\times v))\times e,v}\backslash \mathsf{NP}))/\mathsf{N}_{1\times(e\to e\times v),e}$$

of them $\rightsquigarrow \lambda G^{1\times(e\to e\times v)\to e\to t}.\lambda i^{1\times(e\to e\times v)}.Gi : (\mathsf{N}_{1\times(e\to e\times v),e})^{\mathsf{NPL}_{1\times(e\to e\times v)}}$

read $\rightsquigarrow$

$$\lambda D^{(e\to(1\times(e\to e\times v))\times e\to v\to t)\to(1\times(e\to e\times v))\times e\to v\to t}.\lambda x^{e}.D(\lambda y.\lambda i.\lambda e.\mathrm{read}'(x,y,e))$$

$$: (\mathsf{S}_{(1\times(e\to e\times v))\times e,v}\backslash \mathsf{NP})/(\mathsf{S}_{(1\times(e\to e\times v))\times e,v}/(\mathsf{S}_{(1\times(e\to e\times v))\times e,v}\backslash \mathsf{NP}))$$

it $\rightsquigarrow \lambda g^{(1\times(e\to e\times v))\times e\to e}.\lambda V^{e\to(1\times(e\to e\times v))\times e\to v\to t}.\lambda i^{(1\times(e\to e\times v))\times e}.V(gi)i$

$$: (\mathsf{S}_{(1\times(e\to e\times v))\times e,v}/(\mathsf{S}_{(1\times(e\to e\times v))\times e,v}\backslash \mathsf{NP}))^{\mathsf{NP}_{(1\times(e\to e\times v))\times e}}$$

$$
\dfrac{\dfrac{\dfrac{\textit{read}}{(S_{\dots}\backslash NP)/(S_{\dots}/(S_{\dots}\backslash NP))}}{(S_{\dots}\backslash NP)^{NP_{\dots}}/(S_{\dots}/(S_{\dots}\backslash NP))^{NP_{\dots}}}\,G \qquad \dfrac{\textit{it}}{(S_{\dots}/(S_{\dots}\backslash NP))^{NP_{\dots}}}}{(S_{1\times(e\to e\times v),v}\backslash NP)^{NP_{(1\times(e\to e\times v))\times e}}}\,>
$$

$$
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\textit{most}}{(S_{\dots}/(S_{\dots}\backslash NP_{\dots}))/N_{\dots}}}{((S_{\dots}/(S_{\dots}\backslash NP\dots)))^{NPL_{\dots}}/(N_{\dots})^{NPL_{\dots}}}\,G \qquad \dfrac{\textit{of them}}{(N_{\dots})^{NPL_{\dots}}}}{(S_{\dots}/(S_{\dots}\backslash NP))^{NPL_{\dots}}}\,>}{(S_{\dots})^{NPL_{\dots}}/(S_{\dots}\backslash NP)}\,X}{((S_{\dots})^{NPL_{\dots}})^{NP_{\dots}}/(S_{\dots}\backslash NP)^{NP_{\dots}}}\,G \qquad \dfrac{\textit{read it}}{(S_{\dots}\backslash NP)^{NP_{\dots}}}}{((S_{1\times(e\to e\times v),e\to v})^{NPL_{1\times(e\to e\times v)}})^{NP_{(1\times(e\to e\times v))\times e}}}\,>
$$

Together

$$\begin{array}{cc} \textit{every student bought a book;} & \textit{most of them read it} \\ \vdots & \vdots \end{array}$$

$$\dfrac{((\mathrm{S}_{\dots})^{\mathrm{NPL}_{\dots}})^{\mathrm{NP}_{\dots}} / ((\mathrm{S}_{\dots})^{\mathrm{NPL}_{\dots}})^{\mathrm{NP}_{\dots}} \qquad ((\mathrm{S}_{\dots})^{\mathrm{NPL}_{\dots}})^{\mathrm{NP}_{\dots}}}{((\mathrm{S}_{1,(e\to e\times v)\times(e\to v)})^{\mathrm{NPL}_{1\times(e\to e\times v)}})^{\mathrm{NP}_{(1\times(e\to e\times v))\times e}}} >$$

$$\dfrac{\dfrac{\dfrac{\begin{array}{c}[\text{close}] \\ \mathrm{S}/\mathrm{S}_{1,\dots}\end{array}}{\mathrm{S}^{\mathrm{NPL}_{\dots}}/(\mathrm{S}_{1,\dots})^{\mathrm{NPL}_{\dots}}}\,G}{(\mathrm{S}^{\mathrm{NPL}_{\dots}})^{\mathrm{NP}_{\dots}}/((\mathrm{S}_{1,\dots})^{\mathrm{NPL}_{\dots}})^{\mathrm{NP}_{\dots}}}\,G \qquad \begin{array}{c}\textit{every student bought a book;} \\ \textit{most of them read it} \\ \vdots \\ ((\mathrm{S}_{1,\dots})^{\mathrm{NPL}_{\dots}})^{\mathrm{NP}_{\dots}}\end{array}}{(\mathrm{S}^{\mathrm{NPL}_{1\times(e\to e\times v)}})^{\mathrm{NP}_{(1\times(e\to e\times v))\times e}}} >$$

Interpretation

With pronouns unresolved:

$$\lambda g^{(1\times(e\to e\times v))\times e\to e}.\lambda G^{1\times(e\to e\times v)\to e\to t}.$$

$$\begin{aligned}\exists W^{(e\to e\times v)\times(e\to v)}.\mathrm{dom}(W_0) = \mathsf{student}' \\ \wedge\ \big(\forall x^e.\mathrm{dom}(W_0)x \to (\mathsf{book}'(W_0x)_0 \wedge \mathsf{buy}'(x, W_0x))\big) \\ \wedge\ \mathrm{dom}(W_1) \subseteq G(*, W_0) \wedge \mathsf{most}'(G(*, W_0))(\mathrm{dom}(W_1)) \\ \wedge\ \forall y^e.\mathrm{dom}(W_1)y \to \mathsf{read}'(y, g((*, W_0), y), W_1y)\end{aligned}$$

Interpretation

With pronouns unresolved:

$$\lambda g^{(1\times(e\to e\times v))\times e\to e}.\lambda G^{1\times(e\to e\times v)\to e\to t}.$$

$$\begin{aligned}\exists W^{(e\to e\times v)\times(e\to v)}.\mathrm{dom}(W_0) = \text{student}' & \\ & \wedge \left(\forall x^e.\mathrm{dom}(W_0)x \to (\text{book}'(W_0x)_0 \wedge \text{buy}'(x, W_0x))\right) \\ & \wedge \mathrm{dom}(W_1) \subseteq G(*, W_0) \wedge \text{most}'(G(*, W_0))(\mathrm{dom}(W_1)) \\ & \wedge \forall y^e.\mathrm{dom}(W_1)y \to \text{read}'(y, g((*, W_0), y), W_1y)\end{aligned}$$

Resolution for *it*:

$$\lambda i^{(1\times(e\to e\times v))\times e}.((i_0)_1\, i_1)_0$$

Interpretation

With pronouns unresolved:

$$\lambda g^{(1\times(e\to e\times v))\times e\to e}.\lambda G^{1\times(e\to e\times v)\to e\to t}.$$

$$\begin{aligned}\exists W^{(e\to e\times v)\times(e\to v)}.\mathrm{dom}(W_0) = \text{student}' \\ \wedge \left(\forall x^e.\mathrm{dom}(W_0)x \to (\text{book}'(W_0x)_0 \wedge \text{buy}'(x, W_0x))\right) \\ \wedge\, \mathrm{dom}(W_1) \subseteq G(*, W_0) \wedge \text{most}'(G(*, W_0))(\mathrm{dom}(W_1)) \\ \wedge\, \forall y^e.\mathrm{dom}(W_1)y \to \text{read}'(y, g((*, W_0), y), W_1y)\end{aligned}$$

Resolution for *it*:

$$\lambda i^{(1\times(e\to e\times v))\times e}.((i_0)_1\, i_1)_0$$

Resolution for *of them*:

$$\lambda j^{1\times(e\to e\times v)}.\mathrm{dom}(j_1)$$

Resolved

$$\begin{aligned}\exists W^{(e\to e\times v)\times(e\to v)}.\mathrm{dom}(W_0) &= \mathsf{student}' \\ &\wedge \left(\forall x^e.\mathrm{dom}(W_0)x \to (\mathsf{book}'(W_0x)_0 \wedge \mathsf{buy}'(x, W_0x))\right) \\ &\wedge \mathrm{dom}(W_1) \subseteq \mathrm{dom}(W_0) \wedge \mathsf{most}'(\mathrm{dom}(W_0))(\mathrm{dom}(W_1)) \\ &\wedge \forall y^e.\mathrm{dom}(W_1)y \to \mathsf{read}'(y, (W_0y)_0, W_1y)\end{aligned}$$

Resolved

$$\begin{aligned}\exists W^{(e\to e\times v)\times(e\to v)}.\mathrm{dom}(W_0) = \mathrm{student}' \\ &\wedge \big(\forall x^e.\mathrm{dom}(W_0)x \to (\mathrm{book}'(W_0x)_0 \wedge \mathrm{buy}'(x, W_0x))\big) \\ &\wedge \mathrm{dom}(W_1) \subseteq \mathrm{dom}(W_0) \wedge \mathrm{most}'(\mathrm{dom}(W_0))(\mathrm{dom}(W_1)) \\ &\wedge \forall y^e.\mathrm{dom}(W_1)y \to \mathrm{read}'(y, (W_0y)_0, W_1y)\end{aligned}$$

$$\begin{aligned}\equiv \exists f^{e\to e\times v}.\exists P^{e\to t}.\big(\forall x^e.\mathrm{student}'x \to (\mathrm{book}'(fx)_0 \wedge \mathrm{buy}'(x, fx))\big) \\ &\wedge P \subseteq \mathrm{student}' \wedge \mathrm{most}'\mathrm{student}'P \\ &\wedge \forall y^e.Py \to \exists e^v.\mathrm{read}'(y, (fy)_0, e)\end{aligned}$$

Natural resolution functions (NRFs)

The set of NRFs is the smallest set such that, for any types α, β and γ and any terms $F :: \alpha \to \beta \to \gamma$, $G :: \beta \to \gamma$ and $H :: \alpha \to \beta$:

- $\lambda a^{\alpha}.a$ is an NRF
- $\lambda A^{\alpha \times \beta}.A_0$ is an NRF
- $\lambda A^{\alpha \times \beta}.A_1$ is an NRF
- $\lambda X^{\alpha \times \beta \to t}.\lambda a^{\alpha}.\exists b^{\beta}.X(a, b)$ is an NRF
- $\lambda X^{\alpha \times \beta \to t}.\lambda b^{\alpha}.\exists a^{\beta}.X(a, b)$ is an NRF
- $\lambda f^{\alpha \to \beta}.\mathrm{dom} f$ is an NRF
- $\lambda f^{\alpha \to \beta}.\lambda b^{\beta}.\exists a^{\alpha}.\mathrm{dom} fa \wedge b = fa$ is an NRF
- $\lambda a^{\alpha}.G(Ha)$ is an NRF if G and H are NRFs
- $\lambda a^{\alpha}.Fa(Ha)$ is an NRF if F and H are NRFs

A resolution function can select projections, sets of projections, the domain or range of a function, and can apply one thing it selects to another.

(3) Every player chooses a pawn. He puts it on square one.

(3) Every player chooses a pawn. He puts it on square one.

- In order to deal with examples like (3), Roberts (1987) posits the existence of a covert adverbial at the start of the second sentence, meaning something like ‘in every case’.

(3) Every player chooses a pawn. He puts it on square one.

- In order to deal with examples like (3), Roberts (1987) posits the existence of a covert adverbial at the start of the second sentence, meaning something like ‘in every case’.
- I adopt essentially the same strategy: a silent subordinating operator that, when applied to the usual sentential conjunction (;), gives an alternative, subordinating, sentential conjunction $;_{\text{sub}}$

(3) Every player chooses a pawn. He puts it on square one.

- In order to deal with examples like (3), Roberts (1987) posits the existence of a covert adverbial at the start of the second sentence, meaning something like 'in every case'.
- I adopt essentially the same strategy: a silent subordinating operator that, when applied to the usual sentential conjunction (;), gives an alternative, subordinating, sentential conjunction $;_{\text{sub}}$

$$;_{\mathrm{sub}} \rightsquigarrow \lambda p^{\alpha\to(\beta\to\gamma)\to t}.\lambda q^{\alpha\times\beta\times(\beta\to\gamma)\to\delta\to t}.\lambda i^{\alpha}.\lambda o^{(\beta\to\gamma)\times(\beta\to\delta)}.pio_0$$
$$\wedge\, \mathrm{dom}(o_0) = \mathrm{dom}(o_1) \wedge \forall b^{\beta}.\mathrm{dom}(o_1)b \to q(i, b, o_0)(o_1 b)$$
$$: (\mathsf{s}_{\alpha,(\beta\to\gamma)\times(\beta\to\delta)}/\mathsf{s}_{\alpha\times\beta\times(\beta\to\gamma),\delta})\backslash\mathsf{s}_{\alpha,\beta\to\gamma}$$

(3) Every player chooses a pawn; he puts it on square one.

(3) Every player chooses a pawn; he puts it on square one.

Resolved lexical entries:

(3) Every player chooses a pawn; he puts it on square one.

Resolved lexical entries:

$$every \rightsquigarrow \lambda P^{1\to e\to t}.\lambda V^{e\to 1\times e\to e\times v\to t}.\lambda i^{1}.\lambda f^{e\to e\times v}.\mathrm{dom}f = (Pi) \wedge \forall x^{e}.\mathrm{dom}fx \to Vx(i,x)(fx)$$

$$: (\mathrm{S}_{1,e\to e\times v}/(\mathrm{S}_{1\times e,e\times v}\backslash \mathrm{NP}))/\mathrm{N}_{1,e}$$

$player \rightsquigarrow \lambda i^{1}.\mathrm{player}' : \mathrm{N}_{1,e}$ $\quad$ $pawn \rightsquigarrow \lambda i^{1\times e}.\mathrm{pawn}' : \mathrm{N}_{1\times e,e}$

$$chooses \rightsquigarrow \lambda D^{(e\to(1\times e)\times e\to v\to t)\to 1\times e\to e\times v\to t}.\lambda x^{e}.D(\lambda y^{e}.\lambda i^{(1\times e)\times e}.\lambda e^{v}.\mathrm{choose}'(x,y,e))$$

$$: (\mathrm{S}_{1\times e,e\times v}\backslash \mathrm{NP})/(\mathrm{S}_{1\times e,e\times v}/(\mathrm{S}_{(1\times e)\times e,v}\backslash \mathrm{NP}))$$

$$a \rightsquigarrow \lambda P^{1\times e\to e\to t}.\lambda V^{e\to(1\times e)\times e\to v\to t}.\lambda i^{1\times e}.\lambda u^{e\times v}.Piu_0 \wedge V(u_0)_0(i,u_0)u_1$$

$$: (\mathrm{S}_{1\times e,e\times v}/(\mathrm{S}_{(1\times e)\times e,v}\backslash \mathrm{NP}))/\mathrm{N}_{1\times e,e}$$

$$;_{\mathrm{sub}} \rightsquigarrow \lambda p^{1\to(e\to e\times v)\to t}.\lambda q^{1\times e\times(e\to e\times v)\to v\to t}.\lambda i^{1}.\lambda o^{(e\to e\times v)\times(e\to v)}.pio_0 \wedge \mathrm{dom}o_0 = \mathrm{dom}o_1 \wedge \forall b^{e}.\mathrm{dom}o_1 b \to q(i,b,o_0)(o_1 b)$$

$$: (\mathrm{S}_{1,(e\to e\times v)\times(e\to v)}/\mathrm{S}_{1\times e\times(e\to e\times v),v})\backslash \mathrm{S}_{1,e\to e\times v}$$

First sentence derivation

$$
\dfrac{\dfrac{\dfrac{\overset{\substack{\textit{every player}\\ \textit{chooses a pawn}\\ \vdots}}{\mathsf{S}_{1,e\to e\times v}} \qquad \overset{;\text{sub}}{\left(\mathsf{S}_{1,(e\to e\times v)\times(e\to v)} / \mathsf{S}_{1\times e\times(e\to e\times v),v}\right)\backslash \mathsf{S}_{1,e\to e\times v}}}{\mathsf{S}_{1,(e\to e\times v)\times(e\to v)} / \mathsf{S}_{1\times e\times(e\to e\times v),v}} <}{\left(\mathsf{S}_{1,(e\to e\times v)\times(e\to v)}\right)^{\mathsf{NP}_{1\times e\times(e\to e\times v)}} / \left(\mathsf{S}_{1\times e\times(e\to e\times v),v}\right)^{\mathsf{NP}_{1\times e\times(e\to e\times v)}}} G}{\left(\left(\mathsf{S}_{1,(e\to e\times v)\times(e\to v)}\right)^{\mathsf{NP}_{1\times e\times(e\to e\times v)}}\right)^{\mathsf{NP}_{1\times e\times(e\to e\times v)}} / \left(\left(\mathsf{S}_{1\times e\times(e\to e\times v),v}\right)^{\mathsf{NP}_{1\times e\times(e\to e\times v)}}\right)^{\mathsf{NP}_{1\times e\times(e\to e\times v)}}} G
$$

(3) Every player chooses a pawn; he puts it on square one.

(3) Every player chooses a pawn; he puts it on square one.

Resolved lexical entries:

(3) Every player chooses a pawn; he puts it on square one.

Resolved lexical entries:

$$he \rightsquigarrow \lambda g^{1\times e\times(e\to e\times v)\to e}.\lambda V^{e\to(1\times(e\to e\times v))\times e\to v\to t}.\lambda i^{(1\times(e\to e\times v))\times e}.V(gi)i$$

$$: (\mathrm{S}_{1\times e\times(e\to e\times v),v}/(\mathrm{S}_{1\times e\times(e\to e\times v),v}\backslash\mathrm{NP}))^{\mathrm{NP}_{1\times e\times(e\to e\times v)}}$$

puts ... on square one $\rightsquigarrow$

$$\lambda D^{(e\to 1\times e\times(e\to e\times v)\to v\to t)\to 1\times e\times(e\to e\times v)\to v\to t}.$$

$$\lambda x^{e}.D(\lambda y^{e}.\lambda i^{1\times e\times(e\to e\times v)}.\lambda e^{v}.\mathrm{put}'(x,y,\mathrm{onsq1}',e))$$

$$: (\mathrm{S}_{1\times e\times(e\to e\times v),v}\backslash\mathrm{NP})/(\mathrm{S}_{1\times e\times(e\to e\times v),v}/(\mathrm{S}_{1\times e\times(e\to e\times v),v}\backslash\mathrm{NP}))$$

$$it \rightsquigarrow \lambda g^{1\times e\times(e\to e\times v)\to e}.\lambda V^{e\to(1\times(e\to e\times v))\times e\to v\to t}.\lambda i^{(1\times(e\to e\times v))\times e}.V(gi)i$$

$$: (\mathrm{S}_{1\times e\times(e\to e\times v),v}/(\mathrm{S}_{1\times e\times(e\to e\times v),v}\backslash\mathrm{NP}))^{\mathrm{NP}_{1\times e\times(e\to e\times v)}}$$

Second sentence derivation

$$
\dfrac{
\dfrac{
\dfrac{\dfrac{\textit{he}}{(\mathrm{S}_{\ldots}/(\mathrm{S}_{\ldots}\backslash \mathrm{NP}))^{\mathrm{NP}_{\ldots}}}}{(\mathrm{S}_{\ldots})^{\mathrm{NP}_{\ldots}}/(\mathrm{S}_{\ldots}\backslash \mathrm{NP})}\,X
}{((\mathrm{S}_{\ldots})^{\mathrm{NP}_{\ldots}})^{\mathrm{NP}_{\ldots}}/(\mathrm{S}_{\ldots}\backslash \mathrm{NP})^{\mathrm{NP}_{\ldots}}}\,G
\qquad
\dfrac{
\dfrac{\dfrac{\textit{puts...}}{(\mathrm{S}_{\ldots}\backslash \mathrm{NP})/(\mathrm{S}_{\ldots}/(\mathrm{S}_{\ldots}\backslash \mathrm{NP}))}}{(\mathrm{S}_{\ldots}\backslash \mathrm{NP})^{\mathrm{NP}_{\ldots}}/(\mathrm{S}_{\ldots}/(\mathrm{S}_{\ldots}\backslash \mathrm{NP}))^{\mathrm{NP}_{\ldots}}}\,G
\qquad
\begin{array}{c}\textit{it}\\ \vdots \\ (\mathrm{S}_{\ldots}/(\mathrm{S}_{\ldots}\backslash \mathrm{NP}))^{\mathrm{NP}_{\ldots}}\end{array}
}{(\mathrm{S}_{\ldots}\backslash \mathrm{NP})^{\mathrm{NP}_{\ldots}}}\,>
}{((\mathrm{S}_{1\times e\times(e\to e\times v),v})^{\mathrm{NP}_{1\times e\times(e\to e\times v)}})^{\mathrm{NP}_{1\times e\times(e\to e\times v)}}}\,>
$$

Together

every player chooses a pawn;$_{\text{sub}}$

$$\vdots$$

$$\dfrac{\dfrac{\dfrac{S_{\ldots}/S_{\ldots}}{(S_{\ldots})^{NP_{\ldots}}/(S_{\ldots})^{NP_{\ldots}}}\,G}{((S_{\ldots})^{NP_{\ldots}})^{NP_{\ldots}}/((S_{\ldots})^{NP_{\ldots}})^{NP_{\ldots}}}\,G \qquad \begin{array}{c}\textit{he puts it}\\ \textit{on square one}\\ \vdots \\ ((S_{\ldots})^{NP_{\ldots}})^{NP_{\ldots}}\end{array}}{((S_{1,(e\to e\times v)\times(e\to v)})^{NP_{1\times e\times(e\to e\times v)}})^{NP_{1\times e\times(e\to e\times v)}}} >$$

$$\vdots \text{ [close]}$$

$$(S^{NP_{1\times e\times(e\to e\times v)}})^{NP_{1\times e\times(e\to e\times v)}}$$

Interpretation

With pronouns unresolved:

$$\lambda g^{1\times e\times(e\to e\times v)\to e}.\lambda h^{1\times e\times(e\to e\times v)\to e}.$$

$$\begin{aligned}\exists o^{(e\to e\times v)\times(e\to v)}.\mathrm{dom}(o_0) = \mathsf{player}' \\ \wedge (\forall x^e.\mathrm{dom}(o_0)x \to (\mathsf{pawn}'(o_0x)_0 \wedge \mathsf{choose}'(x, o_0x))) \\ \wedge \mathrm{dom}(o_1) = \mathrm{dom}(o_0) \\ \wedge \forall y^e.\mathrm{dom}(o_1)y \to \mathsf{put}'(h(*, y, o_0), g(*, y, o_0), \mathsf{onsq1}', o_1y)\end{aligned}$$

Interpretation

With pronouns unresolved:

$$\lambda g^{1\times e\times(e\to e\times v)\to e}.\lambda h^{1\times e\times(e\to e\times v)\to e}.$$

$$\begin{aligned}\exists o^{(e\to e\times v)\times(e\to v)}.\mathrm{dom}(o_0) = \mathsf{player}' \\ \wedge (\forall x^e.\mathrm{dom}(o_0)x \to (\mathsf{pawn}'(o_0x)_0 \wedge \mathsf{choose}'(x, o_0x))) \\ \wedge \mathrm{dom}(o_1) = \mathrm{dom}(o_0) \\ \wedge \forall y^e.\mathrm{dom}(o_1)y \to \mathsf{put}'(h(*, y, o_0), g(*, y, o_0), \mathsf{onsq1}', o_1y)\end{aligned}$$

Resolution for *it*:

$$\lambda i^{1\times e\times(e\to e\times v)}.((i_1)_1\,(i_1)_0)_0$$

Interpretation

With pronouns unresolved:

$$\lambda g^{1\times e\times(e\to e\times v)\to e}.\lambda h^{1\times e\times(e\to e\times v)\to e}.$$

$$\begin{aligned}\exists o^{(e\to e\times v)\times(e\to v)}.\mathrm{dom}(o_0) = \mathsf{player}' \\ \wedge (\forall x^e.\mathrm{dom}(o_0)x \to (\mathsf{pawn}'(o_0x)_0 \wedge \mathsf{choose}'(x, o_0x))) \\ \wedge \mathrm{dom}(o_1) = \mathrm{dom}(o_0) \\ \wedge \forall y^e.\mathrm{dom}(o_1)y \to \mathsf{put}'(h(*, y, o_0), g(*, y, o_0), \mathsf{onsq1}', o_1y)\end{aligned}$$

Resolution for *it*:

$$\lambda i^{1\times e\times(e\to e\times v)}.((i_1)_1\,(i_1)_0)_0$$

Resolution for *he*:

$$\lambda j^{1\times e\times(e\to e\times v)}.(j_1)_0$$

Resolved

$$\begin{aligned}\exists o^{(e\to e\times v)\times(e\to v)}.\mathrm{dom}\, o_0 = \mathsf{player}' \\ \wedge (\forall x^e.\mathrm{dom}(o_0)x \to (\mathsf{pawn}'(o_0x)_0 \wedge \mathsf{choose}'(x, o_0x))) \\ \wedge\, \mathrm{dom}(o_1) = \mathrm{dom}(o_0) \\ \wedge\, \forall y^e.\mathrm{dom}(o_1)y \to \mathsf{put}'(y, (o_0y)_0, \mathsf{onsq1}', o_1y)\end{aligned}$$

Resolved

$$\exists o^{(e\to e\times v)\times(e\to v)}.\mathrm{dom} o_0 = \mathsf{player}' \\ \wedge (\forall x^e.\mathrm{dom}(o_0)x \to (\mathsf{pawn}'(o_0 x)_0 \wedge \mathsf{choose}'(x, o_0 x))) \\ \wedge \mathrm{dom}(o_1) = \mathrm{dom}(o_0) \\ \wedge \forall y^e.\mathrm{dom}(o_1)y \to \mathsf{put}'(y, (o_0 y)_0, \mathsf{onsq1}', o_1 y)$$

$$\equiv \exists f^{e\to e\times v}.(\forall x^e.\mathsf{player}' x \to (\mathsf{pawn}'(fx)_0 \wedge \mathsf{choose}'(x, fx))) \\ \wedge \forall y^e.\mathsf{player}' y \to \exists e^v.\mathsf{put}'(y, (fy)_0, \mathsf{onsq1}', e)$$

Varieties of subordinating conjunction

(4) Every player chooses a pawn. He always/usually/rarely[1]/...puts it on square one.

[1] Extra statements are required for non-monotone-increasing quantifiers

(4) Every player chooses a pawn. He always/usually/rarely[1]/...puts it on square one.

Overt subordinating conjunction:

$$\lambda p^{\alpha\to(\beta\to\gamma)\to t}.\lambda q^{\alpha\times\beta\times(\beta\to\gamma)\to\delta\to t}.\lambda i^{\alpha}.\lambda o^{(\beta\to\gamma)\times(\beta\to\delta)}.pio_0$$
$$\wedge\ \mathrm{dom}(o_1)\subseteq\mathrm{dom}(o_0)\wedge \mathit{det}'(\mathrm{dom}(o_0))(\mathrm{dom}(o_1))$$
$$\wedge\ \forall b^{\beta}.\mathrm{dom}(o_1)b\to q(i,b,o_0)(o_1 b)$$

Where det′ can be every′, most′, few′...

[1] Extra statements are required for non-monotone-increasing quantifiers

Discussion

A recap

(2) Every student bought a book. Most of them read it.

$$\lambda g^{(1\times(e\to e\times v))\times e\to e}.\lambda G^{1\times(e\to e\times v)\to e\to t}.$$

$$\begin{aligned}\exists W^{(e\to e\times v)\times(e\to v)}.\mathrm{dom}(W_0) = \text{student}' \\ \wedge\ (\forall x^e.\mathrm{dom}(W_0)x \to (\text{book}'(W_0x)_0 \wedge \text{buy}'(x, W_0x))) \\ \wedge\ \mathrm{dom}(W_1) \subseteq G(*, W_0) \wedge \text{most}'(G(*, W_0))(\mathrm{dom}(W_1)) \\ \wedge\ \forall y^e.\mathrm{dom}(W_1)y \to \text{read}'(y, g((*, W_0), y), W_1y)\end{aligned}$$

A recap

(2) Every student bought a book. Most of them read it.

$$\lambda g^{(1\times(e\to e\times v))\times e\to e}.\lambda G^{1\times(e\to e\times v)\to e\to t}.$$

$$\exists W^{(e\to e\times v)\times(e\to v)}.\mathrm{dom}(W_0) = \text{student}'$$

$$\wedge\ (\forall x^e.\mathrm{dom}(W_0)x \to (\text{book}'(W_0x)_0 \wedge \text{buy}'(x, W_0x)))$$

$$\wedge\ \mathrm{dom}(W_1) \subseteq G(*, W_0) \wedge \text{most}'(G(*, W_0))(\mathrm{dom}(W_1))$$

$$\wedge\ \forall y^e.\mathrm{dom}(W_1)y \to \text{read}'(y, g((*, W_0), y), W_1y)$$

Resolution for *of them* in this system:

$$\lambda j^{1\times(e\to e\times v)}.\mathrm{dom}(j_1) \text{ applied to } (*, W_0) \Rightarrow_\beta \mathrm{dom}(W_0) \quad (= \text{student}')$$

$::1\times(e\to e\times v)\to e\to t$ $\qquad$ $::1\times(e\to e\times v)$

$$\lambda c^{\gamma}.(\Sigma f : (\Pi v : (\Sigma x : e)\textsc{student}(x))$$
$$(\Sigma u : (\Sigma y : e)\textsc{book}(y))\textsc{buy}(v_0, (u_0)_0))$$
$$\mathit{Most}(\lambda x.(@_i : \ldots)(c, f)(x))$$
$$(\lambda x.(@_i : \ldots)(c, f)(x) \times \textsc{read}(x, (@_j : \ldots)((c, f), x)))$$

$$\lambda c^{\gamma}.(\Sigma f : (\Pi v : (\Sigma x : e)\textsc{student}(x)) \\ (\Sigma u : (\Sigma y : e)\textsc{book}(y))\textsc{buy}(v_0, (u_0)_0)) \\ \mathit{Most}(\lambda x.(@_i : \ldots)(c, f)(x)) \\ (\lambda x.(@_i : \ldots)(c, f)(x) \times \textsc{read}(x, (@_j : \ldots)((c, f), x)))$$

Resolution for *of them* in this system:

$$@_i : \gamma \times \begin{pmatrix} (\Pi v : (\Sigma x : e)\textsc{student}(x)) \\ (\Sigma u : (\Sigma y : e)\textsc{book}(y))\textsc{buy}(v_0, (u_0)_0) \end{pmatrix} \to e \to \textbf{type}$$

applied to $(c, f) \Rightarrow_{\beta}$ STUDENT

What could $@_i$ be? It seems that TTS needs an equivalent of dom to make this work, and it's not obvious how to add it.

Final thoughts

- Many examples of anaphoric dependencies look like they depend on functional relationships established in discourse.

Final thoughts

- Many examples of anaphoric dependencies look like they depend on functional relationships established in discourse.
- We have shown that progress in capturing those anaphoric dependencies can be made by taking that impression seriously, i.e. by having sentences denote functions and allowing those functions to serve as pronominal antecedents.

Final thoughts

- Many examples of anaphoric dependencies look like they depend on functional relationships established in discourse.
- We have shown that progress in capturing those anaphoric dependencies can be made by taking that impression seriously, i.e. by having sentences denote functions and allowing those functions to serve as pronominal antecedents.
- We hope to have shown that this is a viable alternative to placeholders like sets of assignment functions.

Final thoughts

- Many examples of anaphoric dependencies look like they depend on functional relationships established in discourse.
- We have shown that progress in capturing those anaphoric dependencies can be made by taking that impression seriously, i.e. by having sentences denote functions and allowing those functions to serve as pronominal antecedents.
- We hope to have shown that this is a viable alternative to placeholders like sets of assignment functions.
- Further work:
  - ‘Paycheck’ pronouns.
  - Modal subordination.

Thanks!

This research is funded by the

LEVERHULME TRUST

Full(er) details

Mini lexicon

input (left context), output (witness)

$$student \rightsquigarrow \lambda i^{\alpha}.\lambda v^{e\times 1}.\mathrm{student}' v_0 : \mathsf{N}_{\alpha,e\times 1}$$

$$a \rightsquigarrow \lambda P^{\alpha\to e\times\beta\to t}.\lambda V^{e\to\alpha\times e\times\beta\to\gamma\to t}.\lambda i^{\alpha}.\lambda u^{(e\times\beta)\times\gamma}.Piu_0 \wedge V(u_0)_0(i,u_0)u_1$$
$$: (\mathsf{S}_{\alpha,(e\times\beta)\times\gamma}/(\mathsf{S}_{\alpha\times e\times\beta,\gamma}\backslash \mathsf{NP}))/\mathsf{N}_{\alpha,e\times\beta}$$

$$who \rightsquigarrow \lambda V^{e\to\alpha\times e\times\beta\to\gamma\to t}.\lambda P^{\alpha\to e\times\beta\to t}.$$
$$\lambda i^{\alpha}.\lambda o^{e\times\beta\times\gamma}.Pi(o_0,(o_1)_0) \wedge Vo_0(i,(o_1)_0)(o_1)_1$$
$$: (\mathsf{N}_{\alpha,e\times\beta\times\gamma}\backslash \mathsf{N}_{\alpha,e\times\beta})/(\mathsf{S}_{\alpha\times e\times\beta,\gamma}\backslash \mathsf{NP})$$

$$det_{\text{weak}/\boxed{\text{strong}}} \rightsquigarrow$$

$$\lambda P^{\alpha\to e\times\beta\to t}.\lambda V^{e\to\alpha\times e\times\beta\to\gamma\to t}.$$

$$\lambda i^{\color{blue}\alpha}.\lambda f^{\color{red}e\times\beta\to\gamma}.\mathrm{dom} f \subseteq (\lambda v^{e\times\beta}.Piv)$$

$$\wedge\ \mathrm{det}'(\lambda x^e.\exists b^\beta.Pi(x,b))(\lambda x^e.\exists b^\beta.\mathrm{dom} f(x,b))$$

$$\wedge\ \big(\forall x^e.\forall b^\beta.\mathrm{dom} f(x,b) \to Vx(i,x,b)(f(x,b))\big)$$

$$\boxed{\wedge\ (\forall x^e.\forall b^\beta.(Pi(x,b) \wedge \exists c^\beta.\mathrm{dom} f(x,c)) \to \mathrm{dom} f(x,b))}$$

$$\wedge\ \neg\exists Y^{e\times\beta\to t}.(\lambda x^e.\exists b^\beta.\mathrm{dom} f(x,b)) \subsetneq (\lambda x^e.\exists b^\beta.Y(x,b))$$

$$\wedge\ \forall x^e.\forall b^\beta.Y(x,b) \to (Pi(x,b) \wedge \exists c^\gamma.Vx(i,x,b)c)$$

‘Half the students who borrowed a book returned it’

Weak/ strong interpretation:

$$\exists f^{e\times e\times v\to v}.\mathrm{dom} f \subseteq (\lambda v^{e\times e\times v}.\mathrm{stdnt}' v_0 \wedge \mathrm{bk}'(v_1)_0 \wedge \mathrm{brrw}'(v_0,(v_1)_0,(v_1)_1))$$

$$\wedge\ \mathrm{half}'(\lambda x^e.\exists u^{e\times v}.\mathrm{stdnt}' x \wedge \mathrm{bk}' u_0 \wedge \mathrm{brrow}'(x,u))(\lambda x^e.\exists u^{e\times v}.\mathrm{dom} f(x,u))$$

$$\wedge\ \big(\forall x^e.\forall u^{e\times v}.\mathrm{dom} f(x,u) \to \mathrm{rtrn}'(x,u_0,f(x,u))\big)$$

$$\boxed{\begin{aligned}\wedge\ \big(\forall x^e.\forall u^{e\times v}.(\mathrm{stdnt}' x \wedge \mathrm{bk}' u_0 \wedge \mathrm{brrw}'(x,u) \wedge \exists c^{e\times v}.\mathrm{dom} f(x,c))\big)\\ \to \mathrm{dom} f(x,u)\end{aligned}}$$

$$\wedge\ \neg\exists Y^{e\times e\times v\to t}.(\lambda x^e.\exists u^{e\times v}.\mathrm{dom} f(x,u)) \subsetneq (\lambda x^e.\exists u^{e\times v}.Y(x,u))$$

$$\wedge\ \forall x^e.\forall u^{e\times v}.Y(x,u) \to \big(\mathrm{stdnt}' x \wedge \mathrm{bk}' u_0 \wedge \mathrm{brrw}'(x,u)$$

$$\wedge\ \exists e^v.\mathrm{rtrn}'(x,u_0,e)\big)$$

More on telescoping

The subordinating conjunction(s) can be seen as the result of applying this function to the standard conjunction *;*:

$$\lambda C^{(\alpha\to(\beta\to\gamma)\to t)\to(\alpha\times(\beta\to\gamma)\to(\beta\to\delta)\to t)\to\alpha\to(\beta\to\gamma)\times(\beta\to\delta)\to t}.$$

$$\lambda p^{\alpha\to(\beta\to\gamma)\to t}.\lambda q^{\alpha\times\beta\times(\beta\to\gamma)\to\delta\to t}.$$

$$\begin{aligned} Cp(\lambda o^{\alpha\times(\beta\to\gamma)}.\lambda f^{\beta\to\gamma}.\mathrm{dom}f \subseteq \mathrm{dom}(o_1) \wedge \det{}'(\mathrm{dom}(o_1))(\mathrm{dom}f)) \\ \wedge \left(\forall b^{\beta}.\mathrm{dom}fb \to q(o_0, b, o_1)(fb)\right) \\ \wedge \neg\exists X^{\beta\to t}.\mathrm{dom}f \subsetneq X \\ \wedge \forall b^{\beta}.Xb \to (\mathrm{dom}(o_1)y \wedge \mathrm{dom}fy) \end{aligned}$$

A fuller statement of the TTS account

$$\lambda c^{\gamma}.(\Sigma f : (\Pi v : (\Sigma x : e)\textsc{student}(x))\; (\Sigma u : (\Sigma y : e)\textsc{book}(y))\textsc{buy}(v_0, (u_0)_0))$$
$$Most(\lambda x.\lambda \delta^{\textbf{type}}.\lambda d^{\delta}.(@_i : (\Pi \alpha : \textbf{type})\alpha \to e \to \textbf{type})(\delta)(d)(x))$$
$$(\lambda x.\lambda \delta^{\textbf{type}}.\lambda d^{\delta}.\textsc{read}(x, (@_i : (\Pi \alpha : \textbf{type})\alpha \to e)(\delta)(d)))$$
$$\left(\gamma \times \begin{array}{l}(\Pi v : (\Sigma x : e)\textsc{student}(x)) \\ \qquad (\Sigma u : (\Sigma y : e)\textsc{book}(y))\textsc{buy}(v_0, (u_0)_0)\end{array}\right)(c, f)$$

So $@_i$ (*of them*) is of type $(\Pi\alpha : \textbf{type})\alpha \to e \to \textbf{type}$, and when applied to its type argument (the third argument of *Most* above) the type is as shown on a previous slide.

Thanks!

This research is funded by the

LEVERHULME TRUST

References

Bekki, Daisuke. 2014. Representing anaphora with dependent types. In Nicholas Asher & Sergei Soloviev (eds.), *Logical aspects of computational linguistics* (Lecture Notes in Computer Science 8535), 14–29. Berlin, Heidelberg: Springer. https://doi.org/10.1007/978-3-662-43742-1_2.

Gotham, Matthew. 2018. A model-theoretic reconstruction of type-theoretic semantics for anaphora. In Annie Foret, Reinhard Muskens & Sylvain Pogodalla (eds.), *Formal grammar: FG 2017* (Lecture Notes in Computer Science 10686), 37–53. Berlin, Heidelberg: Springer. https://doi.org/10.1007/978-3-662-56343-4_3.

Groenendijk, Jeroen & Martin Stokhof. 1991. Dynamic predicate logic. *Linguistics and Philosophy* 14(1). 39–100.

Ranta, Aarne. 1994. *Type-theoretical grammar*. (Indices 1). Oxford: Oxford University Press.

Roberts, Craige. 1987. *Modal subordination, anaphora and distributivity*. University of Massachusetts at Amherst dissertation.

Tanaka, Ribeka, Yuki Nakano & Daisuke Bekki. 2014. Constructive generalized quantifiers revisited. In Yukiko Nakano, Ken Satoh & Daisuke Bekki (eds.), *New frontiers in artificial intelligence: JSAI-isAI 2013* (Lecture Notes in Computer Science 8417), 115–124. Cham: Springer. https://doi.org/10.1007/978-3-319-10061-6_8.