Double negation, excluded middle and accessibility in dynamic semantics

Matthew Gotham
University of Oxford

22nd Amsterdam Colloquium,
University of Amsterdam, 20 December 2019

Summary

As is well known…

Summary

As is well known...

- In pretty much all dynamic semantic theories, negating a formula (DRS, file...) results in a **test**: a formula that doesn't introduce any new anaphoric dependencies that can be picked up in subsequent discourse.

Summary

As is well known…

- In pretty much all dynamic semantic theories, negating a formula (DRS, file…) results in a **test**: a formula that doesn't introduce any new anaphoric dependencies that can be picked up in subsequent discourse.
- It follows that indefinites introduced in the scope of negation are inaccessible as antecedents to pronouns outside the scope of negation.

Summary

As is well known…

- In pretty much all dynamic semantic theories, negating a formula (DRS, file…) results in a **test**: a formula that doesn't introduce any new anaphoric dependencies that can be picked up in subsequent discourse.
- It follows that indefinites introduced in the scope of negation are inaccessible as antecedents to pronouns outside the scope of negation.
- In the simplest cases, this is a welcome result. However, the fact that this 'test-making' is irreversible has some unwelcome consequences.

Summary

As is well known...

- In pretty much all dynamic semantic theories, negating a formula (DRS, file...) results in a **test**: a formula that doesn't introduce any new anaphoric dependencies that can be picked up in subsequent discourse.
- It follows that indefinites introduced in the scope of negation are inaccessible as antecedents to pronouns outside the scope of negation.
- In the simplest cases, this is a welcome result. However, the fact that this 'test-making' is irreversible has some unwelcome consequences.

In this paper, I outline a way of making inaccessible indefinites accessible again, inspired by intuitionistic logic.

Outline

Puzzles of accessibility

Outline

Puzzles of accessibility

Double negation and excluded middle

Outline

Outline

Puzzles of accessibility

Double negation and excluded middle

Accounting for the uniqueness effect

Composition

Outline

Puzzles of accessibility

Double negation and excluded middle

Accounting for the uniqueness effect

Composition

Discussion

Puzzles of accessibility

$$
\begin{aligned}
[\![Pt_1 \ldots t_n]\!]^f_M &= \left\{g \mid f = g \;\&\; \left\langle [\![t_1]\!]^g_M, \ldots, [\![t_n]\!]^g_M \right\rangle \in \mathcal{I}(P)\right\} \\
[\![t_1 = t_2]\!]^f_M &= \left\{g \mid f = g \;\&\; [\![t_1]\!]^g_M = [\![t_2]\!]^g_M\right\} \\
[\![\neg\phi]\!]^f_M &= \left\{g \mid f = g \;\&\; [\![\phi]\!]^g_M = \emptyset\right\} \\
[\![\phi \wedge \psi]\!]^f_M &= \left\{h \mid \text{there's a } g : g \in [\![\phi]\!]^f_M \;\&\; h \in [\![\psi]\!]^g_M\right\} \\
[\![\phi \vee \psi]\!]^f_M &= \left\{g \mid f = g \;\&\; [\![\phi]\!]^g_M \cup [\![\psi]\!]^g_M \neq \emptyset\right\} \\
[\![\phi \rightarrow \psi]\!]^f_M &= \left\{g \mid f = g \;\&\; [\![\phi]\!]^g_M \subseteq \left\{h \mid [\![\psi]\!]^h_M \neq \emptyset\right\}\right\} \\
[\![\exists x\phi]\!]^f_M &= \left\{h \mid \text{there's a } g : f[x]g \;\&\; h \in [\![\phi]\!]^g_M\right\} \\
[\![\forall x\phi]\!]^f_M &= \left\{g \mid f = g \;\&\; \{h \mid g[x]h\} \subseteq \left\{h \mid [\![\phi]\!]^h_M \neq \emptyset\right\}\right\}
\end{aligned}
$$

Cross-sentenctial anaphora

(1) John owns a car. It's parked in a weird place.

(2) $\exists x(Cx \wedge Ojx) \wedge Px$

$$\llbracket(2)\rrbracket^f_M = \Big\{h \mid \text{ there's a } g : g \in \llbracket\exists x(Cx \wedge Ojx)\rrbracket^f_M \;\&\; h \in \llbracket Px\rrbracket^g_M\Big\}$$

$$= \{g \mid f[x]g \;\&\; g(x) \in \mathcal{I}(C) \;\&\; \langle\mathcal{I}(j), g(x)\rangle \in \mathcal{I}(O) \;\&\; g(x) \in \mathcal{I}(P)\}$$

Blocked by negation

(3) John doesn't own a car. It's parked in a weird place.

(4) $\neg\exists x(Cx \wedge Ojx) \wedge Px$

$$
\begin{aligned}
[\![(4)]\!]^f_M &= \left\{h \mid \text{ there's a } g : g \in [\![\neg\exists x(Cx \wedge Ojx)]\!]^f_M \;\&\; h \in [\![Px]\!]^g_M\right\} \\
&= \{g \mid f = g \;\&\; [\![\exists x(Cx \wedge Ojx)]\!]^g_M = \emptyset \;\&\; g(x) \in \mathcal{I}(P)\}
\end{aligned}
$$

Negation is not an involution

> *[T]he law of double negation will not hold unconditionally. Consider a formula ϕ that is not a test. Negating ϕ results in the test $\neg\phi$, and a second negation, which gives $\neg\neg\phi$, does not reverse this effect [...] Hence, double negation is not in general eliminable.*

(Groenendijk & Stokhof 1991: 62)

Double negation

(5) It's not true that John doesn't own a car. It's (just) parked in a weird place.

(6) $\neg\neg\exists x(Cx \wedge Ojx) \wedge Px$

$$[\![(6)]\!]^f_M = \left\{ h \mid \text{ there's a } g : g \in [\![\neg\neg\exists x(Cx \wedge Ojx)]\!]^f_M \;\&\; h \in [\![Px]\!]^g_M \right\}$$
$$= \left\{ g \mid f = g \;\&\; [\![\exists x(Cx \wedge Ojx)]\!]^g_M \neq \emptyset \;\&\; g(x) \in \mathcal{I}(P) \right\}$$

Disjunction

(7) Either John doesn't own a car, or it's parked in a weird place.

(8) $\neg\exists x(Cx \wedge Ojx) \vee Px$

$$\llbracket(8)\rrbracket^{f}_{M} = \left\{ g \mid f = g \;\&\; \begin{array}{l} \left\{ h \mid g = h \;\&\; \llbracket\exists x(Cx \wedge Ojx)\rrbracket^{h}_{M} = \emptyset \right\} \\ \cup \left\{ h \mid g = h \;\&\; h(x) \in \mathcal{I}(P) \right\} \end{array} \neq \emptyset \right\}$$
$$= \begin{cases} \{f\} & \text{if } \llbracket\exists x(Cx \wedge Ojx)\rrbracket^{f}_{M} = \emptyset \text{ or } f(x) \in \mathcal{I}(P) \\ \emptyset & \text{otherwise} \end{cases}$$

Why think these issues are related?

Note that in PL, (8) is equivalent to both (9) and (10).

(9) $\neg\exists x(Cx \wedge Ojx) \vee (\exists x(Cx \wedge Ojx) \wedge Px)$

(10) $\neg\exists x(Cx \wedge Ojx) \vee (\neg\neg\exists x(Cx \wedge Ojx) \wedge Px)$

Why think these issues are related?

Note that in PL, (8) is equivalent to both (9) and (10).

(9) $\neg\exists x(Cx \wedge Ojx) \vee (\exists x(Cx \wedge Ojx) \wedge Px)$

(10) $\neg\exists x(Cx \wedge Ojx) \vee (\neg\neg\exists x(Cx \wedge Ojx) \wedge Px)$

- In DPL (8) is equivalent to (10) but not (9); and (9) *would* capture the intended dependency when interpreted in DPL.

Why think these issues are related?

Note that in PL, (8) is equivalent to both (9) and (10).

(9) $\neg\exists x(Cx \wedge Ojx) \vee (\exists x(Cx \wedge Ojx) \wedge Px)$

(10) $\neg\exists x(Cx \wedge Ojx) \vee (\neg\neg\exists x(Cx \wedge Ojx) \wedge Px)$

- In DPL (8) is equivalent to (10) but not (9); and (9) *would* capture the intended dependency when interpreted in DPL.
- So, apparently, we again have a situation where the PL equivalence based on double negation would be desirable.

Uniqueness

However, it seems that we don't want ϕ to be *exactly* equivalent to $\neg\neg\phi$.

Uniqueness

However, it seems that we don't want ϕ to be *exactly* equivalent to $\neg\neg\phi$.

(5) It's not true that John doesn't own a car. It's (just) parked in a weird place.

(11) ??It's not true that John doesn't own a shirt. It's in the wardrobe.

(7) Either John doesn't own a car, or it's parked in a weird place.

(12) ??Either John doesn't own a shirt, or it's in the wardrobe.

These examples seem to carry the implication that, if John owns a car/shirt, then he owns exactly one.

More contrasts

(13) John owns a car. It's parked in a weird place. He owns another one which is in the garage.

(14) ??It's not true that John doesn't own a car. It's just parked in a weird place. He owns another one which is in the garage.

(15) ??Either John doesn't own a car, or it's parked in a weird place and he owns another one which is in the garage.

Uniqueness?

Matt Mandelkern (p.c.) has expressed doubts about uniqueness implications, on the basis of examples like (16).

(16) ?Either Sue didn't have a drink last night, or she had a second drink right after it.

Uniqueness?

Matt Mandelkern (p.c.) has expressed doubts about uniqueness implications, on the basis of examples like (16).

(16) ?Either Sue didn't have a drink last night, or she had a second drink right after it.

- Personally I find this example strange too, but admittedly have an interest in doing so.

Uniqueness?

Matt Mandelkern (p.c.) has expressed doubts about uniqueness implications, on the basis of examples like (16).

(16) ?Either Sue didn't have a drink last night, or she had a second drink right after it.

- Personally I find this example strange too, but admittedly have an interest in doing so.
- In what follows I'll present two accounts, with and without uniqueness implications.

Previous accounts/suggestions

Previous accounts/suggestions

- Decompose negation (Groenendijk & Stokhof 1990, Rothschild 2017)

Previous accounts/suggestions

- Decompose negation (Groenendijk & Stokhof 1990, Rothschild 2017)
- Bilateralism (Krahmer & Muskens 1995)

Previous accounts/suggestions

- Decompose negation (Groenendijk & Stokhof 1990, Rothschild 2017)
- Bilateralism (Krahmer & Muskens 1995)

Neither has much to say about uniqueness implications.

Double negation and excluded middle

DPL and IL

- The non-equivalence of ϕ and $\neg\neg\phi$ in DPL is reminiscent of the situation in intuitionistic logic (IL).

DPL and IL

- The non-equivalence of ϕ and $\neg\neg\phi$ in DPL is reminiscent of the situation in intuitionistic logic (IL).
- The parallel is by no means exact, since in IL this non-equivalence can be expressed as $\phi \dashv\!\!\!/ \vdash \neg\neg\phi$, whereas in DPL it can't really be brought out directly in terms of entailment or derivability.

- The non-equivalence of ϕ and $\neg\neg\phi$ in DPL is reminiscent of the situation in intuitionistic logic (IL).
- The parallel is by no means exact, since in IL this non-equivalence can be expressed as $\phi \not\dashv\vdash \neg\neg\phi$, whereas in DPL it can't really be brought out directly in terms of entailment or derivability.
- Nevertheless, it's worth looking at what one needs to add to IL in order to get the equivalence back.

Intuitionistically unacceptable reasoning

Famously, adding any of (17)–(19) to IL gets you classical logic.

(17) $\neg\neg\phi \vdash \phi$ (double negation elimination)

(18) $$\frac{\Gamma, \neg\phi \vdash \bot}{\Gamma \vdash \phi}$$ (reductio ad absurdum)

(19) $\vdash \phi \vee \neg\phi$ (excluded middle)

Intuitionistically unacceptable reasoning

Famously, adding any of (17)–(19) to IL gets you classical logic.

(17) $\neg\neg\phi \vdash \phi$ (double negation elimination)

(18) $\dfrac{\Gamma, \neg\phi \vdash \bot}{\Gamma \vdash \phi}$ (reductio ad absurdum)

(19) $\vdash \phi \vee \neg\phi$ (excluded middle)

- Question: could there be a way to achieve (something like) the double negation property for dynamic semantics by adding (something like) excluded middle?
- And could that help to resolve the issues we've identified with pronoun accessibility?

Intuitionistically unacceptable reasoning

Famously, adding any of (17)–(19) to IL gets you classical logic.

(17) $\neg\neg\phi \vdash \phi$ (double negation elimination)

(18) $\dfrac{\Gamma, \neg\phi \vdash \bot}{\Gamma \vdash \phi}$ (reductio ad absurdum)

(19) $\vdash \phi \vee \neg\phi$ (excluded middle)

- Question: could there be a way to achieve (something like) the double negation property for dynamic semantics by adding (something like) excluded middle?
- And could that help to resolve the issues we've identified with pronoun accessibility?
- Answer: *yes*, but it doesn't involve the standard DPL disjunction.

Program disjunction (Groenendijk & Stokhof 1991: 88)

$$[\![\phi \cup \psi]\!]^f_M = [\![\phi]\!]^f_M \cup [\![\psi]\!]^f_M$$

Program disjunction (Groenendijk & Stokhof 1991: 88)

$$[\![\phi \cup \psi]\!]^f_M = [\![\phi]\!]^f_M \cup [\![\psi]\!]^f_M$$

- Like $\vee$, $\cup$ is internally static, but unlike $\vee$ it is externally dynamic.

$$[\![\phi \cup \psi]\!]^f_M = [\![\phi]\!]^f_M \cup [\![\psi]\!]^f_M$$

- Like $\vee$, $\cup$ is internally static, but unlike $\vee$ it is externally dynamic.
- In DPL $\phi \cup \neg\phi$ is a tautology — i.e. for any M and f, $[\![\phi \cup \neg\phi]\!]^f_M \neq \emptyset$ — but there are many semantically distinct tautologies in DPL.

Program disjunction (Groenendijk & Stokhof 1991: 88)

$$[\![\phi \cup \psi]\!]^f_M = [\![\phi]\!]^f_M \cup [\![\psi]\!]^f_M$$

- Like $\vee$, $\cup$ is internally static, but unlike $\vee$ it is externally dynamic.
- In DPL $\phi \cup \neg\phi$ is a tautology — i.e. for any M and f, $[\![\phi \cup \neg\phi]\!]^f_M \neq \emptyset$ — but there are many semantically distinct tautologies in DPL.
- Consequently, DPL does not have the property that ϕ is equivalent to $T \wedge \phi$ for any DPL tautology T and formula ϕ.

Program disjunction (Groenendijk & Stokhof 1991: 88)

$$[\![\phi \cup \psi]\!]^f_M = [\![\phi]\!]^f_M \cup [\![\psi]\!]^f_M$$

- Like $\vee$, $\cup$ is internally static, but unlike $\vee$ it is externally dynamic.
- In DPL $\phi \cup \neg\phi$ is a tautology — i.e. for any M and f, $[\![\phi \cup \neg\phi]\!]^f_M \neq \emptyset$ — but there are many semantically distinct tautologies in DPL.
- Consequently, DPL does not have the property that ϕ is equivalent to $T \wedge \phi$ for any DPL tautology T and formula ϕ.
- Relevance for us: the presence of specific tautologies can make inaccessible discourse referents accessible again.

Double negation

For example, if we expand

(6) $\neg\neg\exists x(Cx \wedge Ojx) \wedge Px$

Double negation

For example, if we expand

(6) $\neg\neg\exists x(Cx \wedge Ojx) \wedge Px$

to

(20) $(\exists x(Cx \wedge Ojx) \cup \neg\exists x(Cx \wedge Ojx)) \wedge (\neg\neg\exists x(Cx \wedge Ojx) \wedge Px)$

Double negation

For example, if we expand

(6) $\neg\neg\exists x(Cx \wedge Ojx) \wedge Px$

to

(20) $(\exists x(Cx \wedge Ojx) \cup \neg\exists x(Cx \wedge Ojx)) \wedge (\neg\neg\exists x(Cx \wedge Ojx) \wedge Px)$

we get the binding back:

$$\left[\!\left[(20)\right]\!\right]^f_M = \left\{ g \mid \begin{array}{l} g \in [\![\exists x(Cx \wedge Ojx)]\!]^f_M \cup [\![\neg\exists x(Cx \wedge Ojx)]\!]^f_M \\ \&\ [\![\exists x(Cx \wedge Ojx)]\!]^g_M \neq \emptyset \ \&\ g(x) \in \mathcal{I}(P) \end{array} \right\}$$

$$= \left\{ g \mid g \in [\![\exists x(Cx \wedge Ojx)]\!]^f_M \ \&\ g(x) \in \mathcal{I}(P) \right\}$$

Disjunction

And if we expand

(8) $\neg\exists x(Cx \wedge Ojx) \vee Px$

Disjunction

And if we expand

(8) $\neg\exists x(Cx \wedge Ojx) \vee Px$

to

(21) $(\exists x(Cx \wedge Ojx) \cup \neg\exists x(Cx \wedge Ojx)) \wedge (\neg\exists x(Cx \wedge Ojx) \vee Px)$

Disjunction

And if we expand

(8) $\neg\exists x(Cx \wedge Ojx) \vee Px$

to

(21) $(\exists x(Cx \wedge Ojx) \cup \neg\exists x(Cx \wedge Ojx)) \wedge (\neg\exists x(Cx \wedge Ojx) \vee Px)$

we get a bound reading:

$$\left[\!\left[(21)\right]\!\right]^f_M = \left\{ h \mid \begin{array}{l} \text{there's a } g : g \in [\![\exists x(Cx \wedge Ojx)]\!]^f_M \cup [\![\neg\exists x(Cx \wedge Ojx)]\!]^f_M \\ \& \ h \in [\![\neg\exists x(Cx \wedge Ojx) \vee Px]\!]^g_M \end{array} \right\}$$

$$= \left\{ g \mid \begin{array}{l} (f = g \ \& \ [\![\exists x(Cx \wedge Ojx)]\!]^g_M = \emptyset) \text{ or} \\ \Big(g \in [\![\exists x(Cx \wedge Ojx)]\!]^f_M \ \& \ g(x) \in \mathcal{I}(P)\Big) \end{array} \right\}$$

A harmless addition

Adding excluded middle with $\cup$ doesn't cause a problem in a simple positive example:

$$\text{If } \phi \simeq \phi \wedge \phi \text{ then } (\phi \cup \neg\phi) \wedge \phi \simeq \phi$$

So:

$$(\exists x(Cx \wedge Ojx) \cup \neg\exists x(Cx \wedge Ojx)) \wedge \exists x(Cx \wedge Ojx) \simeq \exists x(Cx \wedge Ojx)$$

A harmless addition

Adding excluded middle with $\cup$ doesn't cause a problem in a simple positive example:

$$\text{If } \phi \simeq \phi \wedge \phi \text{ then } (\phi \cup \neg\phi) \wedge \phi \simeq \phi$$

So:

$$(\exists x(Cx \wedge Ojx) \cup \neg\exists x(Cx \wedge Ojx)) \wedge \exists x(Cx \wedge Ojx) \simeq \exists x(Cx \wedge Ojx)$$

Nor in a single-negation example:

$$\text{If } \phi \simeq \phi \wedge \phi \text{ then } (\phi \cup \neg\phi) \wedge \neg\phi \simeq \neg\phi$$

So:

$$(\exists x(Cx \wedge Ojx) \cup \neg\exists x(Cx \wedge Ojx)) \wedge \neg\exists x(Cx \wedge Ojx) \simeq \neg\exists x(Cx \wedge Ojx)$$

Two features to note

Binding is predicted to be symmetric in disjunction, i.e. (22) is predicted to be just as good as (7).

(7) Either John doesn't own a car, or it's parked in a weird place.

(22) Either it's parked in a weird place, or John doesn't own a car.

Two features to note

Binding is predicted to be symmetric in disjunction, i.e. (22) is predicted to be just as good as (7).

(7) Either John doesn't own a car, or it's parked in a weird place.

(22) Either it's parked in a weird place, or John doesn't own a car.

- In either case the interpretation is 'either John doesn't own a car, or some car he owns is parked in a weird place'.
- This 'weak' take on (7) is disputed by Krahmer & Muskens (1995), who defend a 'strong' reading.

Two features to note

Binding is predicted to be symmetric in disjunction, i.e. (22) is predicted to be just as good as (7).

(7) Either John doesn't own a car, or it's parked in a weird place.

(22) Either it's parked in a weird place, or John doesn't own a car.

- In either case the interpretation is 'either John doesn't own a car, or some car he owns is parked in a weird place'.
- This 'weak' take on (7) is disputed by Krahmer & Muskens (1995), who defend a 'strong' reading.
- The distinction is somewhat moot, though, given the uniqueness effect.

Accounting for the uniqueness effect

What program disjunction does

Let's reflect on what program disjunction does in cases of an existential statement and its negation.

$$[\![\exists xPx \cup \neg\exists xPx]\!]^f_M = \begin{cases} \text{the set of } x\text{-variants of } f \text{ mapping } x \text{ to a } P, \\ \quad \text{if there are any} \\ \{f\} \text{ otherwise} \end{cases}$$

What program disjunction does

Let's reflect on what program disjunction does in cases of an existential statement and its negation.

$$[\![\exists x Px \cup \neg\exists x Px]\!]^f_M = \begin{cases} \text{the set of } x\text{-variants of } f \text{ mapping } x \text{ to a } P, \\ \quad \text{if there are any} \\ \{f\} \text{ otherwise} \end{cases}$$

If we want the anaphoric dependency to be passed on only in the case of uniqueness, then the input context for our unaugmented formulae should look like this instead:

$$\begin{cases} \text{the (singleton) set of } x\text{-variants of } f \text{ mapping } x \text{ to a } P, \\ \quad \text{if there's exactly one} \\ \{f\} \text{ otherwise} \end{cases}$$

That effect can be achieved by the introduction of an operator 1, defined in (23).

(23) $$[\![1\phi]\!]^f_M = \begin{cases} [\![\phi]\!]^f_M & \text{if } \left|[\![\phi]\!]^f_M\right| = 1 \\ \{f\} & \text{otherwise} \end{cases}$$

Or, equivalently,

$$[\![1\phi]\!]^f_M = \left\{g \mid g \in [\![\phi]\!]^f_M \;\&\; \left|[\![\phi]\!]^f_M\right| = 1\right\} \cup \left\{g \mid f = g \;\&\; \left|[\![\phi]\!]^g_M\right| \neq 1\right\}$$

Unique excluded middle

That effect can be achieved by the introduction of an operator $\rceil$, defined in (23).

(23) $$[\![\rceil\phi]\!]^f_M = \begin{cases} [\![\phi]\!]^f_M & \text{if } \left|[\![\phi]\!]^f_M\right| = 1 \\ \{f\} & \text{otherwise} \end{cases}$$

Or, equivalently,

$$[\![\rceil\phi]\!]^f_M = \left\{g \mid g \in [\![\phi]\!]^f_M \;\&\; \left|[\![\phi]\!]^f_M\right| = 1\right\} \cup \left\{g \mid f = g \;\&\; \left|[\![\phi]\!]^g_M\right| \neq 1\right\}$$

- Note that $\rceil\phi$ is also a DPL tautology (for any ϕ).
- I will henceforth refer to formulae of the form $\rceil\phi$ as instances of 'unique excluded middle' (UEM).

Double negation

If, now, we expand

(6) $\neg\neg\exists x(Cx \wedge Ojx) \wedge Px$

Double negation

If, now, we expand

(6) $\neg\neg\exists x(Cx \wedge Ojx) \wedge Px$ to

(24) $\rceil\exists x(Cx \wedge Ojx) \wedge (\neg\neg\exists x(Cx \wedge Ojx) \wedge Px)$

Double negation

If, now, we expand

(6) $\neg\neg\exists x(Cx \wedge Ojx) \wedge Px$ to

(24) $\rceil\exists x(Cx \wedge Ojx) \wedge (\neg\neg\exists x(Cx \wedge Ojx) \wedge Px)$

we get the binding **on the assumption of uniqueness**:

$$\llbracket(24)\rrbracket^f_M = \left\{ g \mid \begin{array}{l} \left(\llbracket\exists x(Cx \wedge Ojx)\rrbracket^f_M = \{g\} \;\&\; g(x) \in \mathcal{I}(P)\right) \\ \text{or } \left(f = g \;\&\; \left|\llbracket\exists x(Cx \wedge Ojx)\rrbracket^f_M\right| > 1 \;\&\; g(x) \in \mathcal{I}(P)\right) \end{array} \right\}$$

'Either John owns exactly one car, which is parked in a weird place, or John owns more than one car and x is parked in a weird place' (with x free).

Disjunction

And if we expand

(8) $\neg\exists x(Cx \wedge Ojx) \vee Px$

And if we expand

(8) $\neg \exists x(Cx \wedge Ojx) \vee Px$ to

(25) $\upharpoonleft \exists x(Cx \wedge Ojx) \wedge (\neg \exists x(Cx \wedge Ojx) \vee Px)$

Disjunction

And if we expand

(8) $\neg\exists x(Cx \wedge Ojx) \vee Px$ to

(25) $\rceil\exists x(Cx \wedge Ojx) \wedge (\neg\exists x(Cx \wedge Ojx) \vee Px)$

we get the binding **on the assumption of uniqueness**:

$$\left[\!\left[(25)\right]\!\right]^f_M = \left\{ g \mid \begin{array}{l} \Big([\![\exists x(Cx \wedge Ojx)]\!]^f_M = \{g\} \;\&\; g(x) \in \mathcal{I}(P) \Big) \\ \text{or } \; \big(f = g \;\&\; [\![\exists x(Cx \wedge Ojx)]\!]^g_M = \emptyset\big) \\ \text{or } \; \Big(f = g \;\&\; \Big|[\![\exists x(Cx \wedge Ojx)]\!]^f_M\Big| > 1 \;\&\; g(x) \in \mathcal{I}(P)\Big) \end{array} \right\}$$

'Either John doesn't own a car, or he owns exactly one car, which is parked in a weird place, or he owns more than one car and x is parked in a weird place' (with x free).

Also a harmless addition

Adding unique excluded middle doesn't cause a problem in a simple positive example either:

$$\text{If } \phi \simeq \phi \wedge \phi \text{ then } \urcorner\phi \wedge \phi \simeq \phi$$

So

$$\urcorner\exists x(Cx \wedge Ojx) \wedge \exists x(Cx \wedge Ojx) \simeq \exists x(Cx \wedge Ojx)$$

Also a harmless addition

Adding unique excluded middle doesn't cause a problem in a simple positive example either:

$$\text{If } \phi \simeq \phi \wedge \phi \text{ then } \rceil\phi \wedge \phi \simeq \phi$$

So

$$\rceil\exists x(Cx \wedge Ojx) \wedge \exists x(Cx \wedge Ojx) \simeq \exists x(Cx \wedge Ojx)$$

Nor in a single-negation example:

$$\text{If } \phi \simeq \phi \wedge \phi \text{ then } \rceil\phi \wedge \neg\phi \simeq \neg\phi$$

So:

$$\rceil\exists x(Cx \wedge Ojx) \wedge \neg\exists x(Cx \wedge Ojx) \simeq \neg\exists x(Cx \wedge Ojx)$$

Composition

An obvious question

Where do the needed instances of EM/UEM come from?

An obvious question

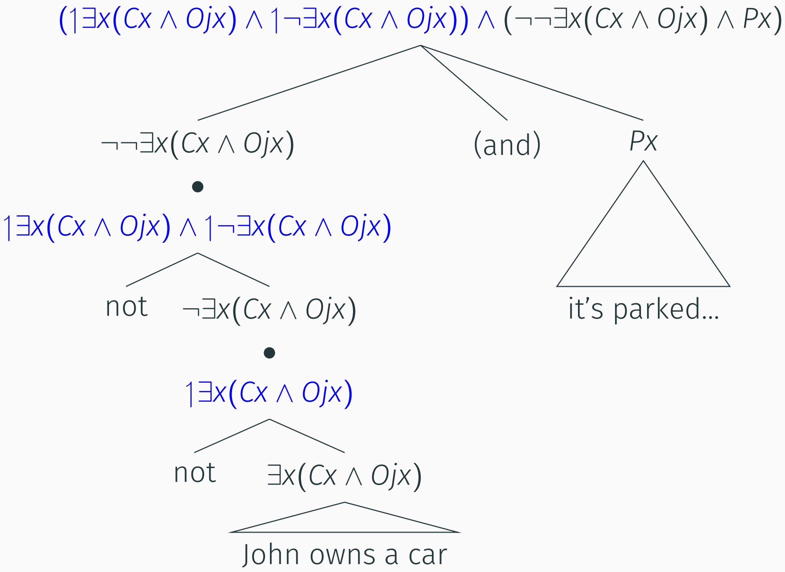
Where do the needed instances of EM/UEM come from? One thought:

- Treat them as introduced lexically by negation as a kind of projective content.

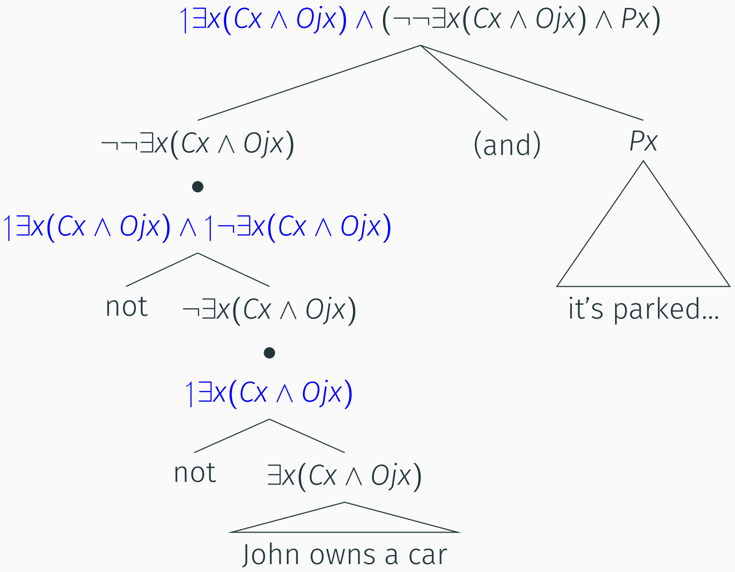
An obvious question

Where do the needed instances of EM/UEM come from? One thought:

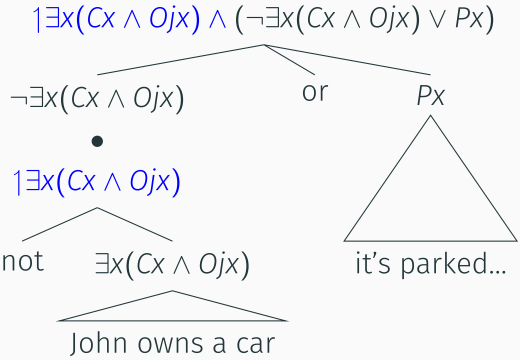
- Treat them as introduced lexically by negation as a kind of projective content.
- Doing this properly requires moving from DPL to a dynamic semantic system that permits compositionality below the level of the clause, so I'll just give a schematic treatment (assuming UEM, and with apologies to Potts (2005)).


(1∃x(Cx ∧ Ojx) ∧ 1¬∃x(Cx ∧ Ojx)) ∧ (¬¬∃x(Cx ∧ Ojx) ∧ Px)
¬¬∃x(Cx ∧ Ojx)
(and)
Px
1∃x(Cx ∧ Ojx) ∧ 1¬∃x(Cx ∧ Ojx)
not
¬∃x(Cx ∧ Ojx)
it's parked...
1∃x(Cx ∧ Ojx)
not
∃x(Cx ∧ Ojx)
John owns a car

Double negation



Disjunction



Discussion

Final thoughts

- On the one hand, the approach in this paper is very conservative about the underlying logic of dynamics. I haven't changed the clauses for any existing DPL connective.

Final thoughts

- On the one hand, the approach in this paper is very conservative about the underlying logic of dynamics. I haven't changed the clauses for any existing DPL connective.
- In particular, I haven't actually made $\vee$ internally dynamic or $\neg$ involutive.

Final thoughts

- On the one hand, the approach in this paper is very conservative about the underlying logic of dynamics. I haven't changed the clauses for any existing DPL connective.
- In particular, I haven't actually made $\vee$ internally dynamic or $\neg$ involutive.
- On the other hand, it requires a novel compositional (or other) story of how these instances of EM/UEM are introduced.

Final thoughts

- On the one hand, the approach in this paper is very conservative about the underlying logic of dynamics. I haven't changed the clauses for any existing DPL connective.
- In particular, I haven't actually made $\vee$ internally dynamic or $\neg$ involutive.
- On the other hand, it requires a novel compositional (or other) story of how these instances of EM/UEM are introduced.
- There's a ready account of the uniqueness effect … if that effect is real.

Thanks!

This research is funded by the

LEVERHULME TRUST

References

- Groenendijk, Jeroen & Martin Stokhof. 1990. *Dynamic Montague Grammar*. Tech. rep. LP-1990-02. ILLC, University of Amsterdam. `https://eprints.illc.uva.nl/1148`.
- Groenendijk, Jeroen & Martin Stokhof. 1991. Dynamic predicate logic. *Linguistics and Philosophy* 14(1). 39–100.
- Krahmer, Emiel & Reinhard Muskens. 1995. Negation and disjunction in discourse representation theory. *Journal of Semantics* 12. 357–376.
- Potts, Christopher. 2005. *The logic of conventional implicatures*. (Oxford Studies in Theoretical Linguistics 7). Oxford: Oxford University Press.

Rothschild, Daniel. 2017. A trivalent approach to anaphora and presupposition. In Alexandre Cremers, Thom van Gessel & Floris Roelofsen (eds.), *Proceedings of the 21st amsterdam colloquium*, 1–13. http://events.illc.uva.nl/AC/AC2017/Proceedings/.